



The CENTRE for EDUCATION  
in MATHEMATICS and COMPUTING  
*cemc.uwaterloo.ca*

# *Galois Contest*

(Grade 10)

**Wednesday, April 13, 2016**  
(in North America and South America)

**Thursday, April 14, 2016**  
(outside of North America and South America)



UNIVERSITY OF  
**WATERLOO**

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**Time:** 75 minutes

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*Do not open this booklet until instructed to do so.*

**Number of questions:** 4

**Each question is worth 10 marks**

**Calculators are allowed, with the following restriction: you may not use a device that has internet access, that can communicate with other devices, or that contains previously stored information. For example, you may not use a smartphone or a tablet.**

Parts of each question can be of two types:

1. **SHORT ANSWER** parts indicated by



- worth 2 or 3 marks each
- full marks given for a correct answer which is placed in the box
- **part marks awarded only if relevant work** is shown in the space provided

2. **FULL SOLUTION** parts indicated by



- worth the remainder of the 10 marks for the question
- **must be written in the appropriate location** in the answer booklet
- marks awarded for completeness, clarity, and style of presentation
- a correct solution poorly presented will not earn full marks

**WRITE ALL ANSWERS IN THE ANSWER BOOKLET PROVIDED.**

- Extra paper for your finished solutions supplied by your supervising teacher must be inserted into your answer booklet. Write your name, school name, and question number on any inserted pages.
- Express calculations and answers as exact numbers such as  $\pi + 1$  and  $\sqrt{2}$ , etc., rather than as  $4.14\dots$  or  $1.41\dots$ , except where otherwise indicated.



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*Do not discuss the problems or solutions from this contest online for the next 48 hours.*

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*The name, grade, school and location of some top-scoring students will be published on our website, [cemc.uwaterloo.ca](http://cemc.uwaterloo.ca). In addition, the name, grade, school and location, and score of some top-scoring students may be shared with other mathematical organizations for other recognition opportunities.*

NOTE:

1. Please read the instructions on the front cover of this booklet.
2. Write all answers in the answer booklet provided.
3. For questions marked , place your answer in the appropriate box in the answer booklet and **show your work**.
4. For questions marked , provide a well-organized solution in the answer booklet. Use mathematical statements and words to explain all of the steps of your solution. Work out some details in rough on a separate piece of paper before writing your finished solution.
5. Diagrams are *not* drawn to scale. They are intended as aids only.
6. While calculators may be used for numerical calculations, other mathematical steps must be shown and justified in your written solutions and specific marks may be allocated for these steps. For example, while your calculator might be able to find the  $x$ -intercepts of the graph of an equation like  $y = x^3 - x$ , you should show the algebraic steps that you used to find these numbers, rather than simply writing these numbers down.
7. No student may write more than one of the Fryer, Galois and Hypatia Contests in the same year.

1. Liza has a row of buckets. The first bucket contains 17 green discs and 7 red discs. Each bucket after the first contains 1 more green disc and 3 more red discs than the previous bucket.



(a) Which bucket contains 16 red discs?

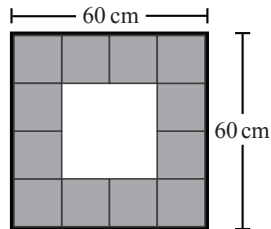


(b) In which bucket is the number of red discs equal to the number of green discs?



(c) There is a bucket in which the number of red discs is twice the number of green discs. In total, how many discs are in this bucket?

2. Judy has square plates, each with side length 60 cm. A plate is *Shanks-Decorated* if identical shaded squares are drawn along the outer edges of the plate, as shown. The diagram shows an example of a plate that is Shanks-Decorated with 12 shaded squares.



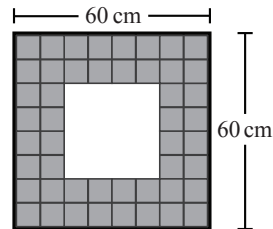
(a) Judy's first plate is Shanks-Decorated with 36 shaded squares. What is the side length of each shaded square?




(b) When a second plate is Shanks-Decorated, an area of  $1600 \text{ cm}^2$  is left unshaded in the centre of the plate. What is the side length of each shaded square?

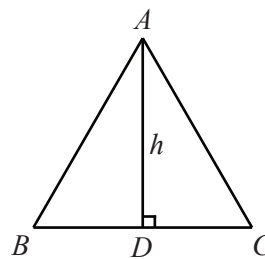



(c) A plate is *Double-Shanks-Decorated* if two layers of identical shaded squares are drawn along the outer edges of the plate, as shown. The diagram shows an example of a plate that is Double-Shanks-Decorated with 48 shaded squares.

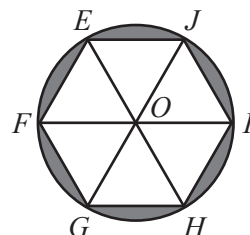



A new plate is Double-Shanks-Decorated and an area of  $2500 \text{ cm}^2$  is left unshaded in the centre of the plate. Determine the number of shaded squares.

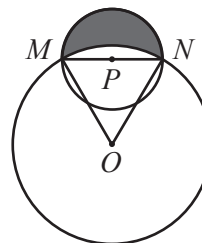
3.  (a) In the diagram,  $\triangle ABC$  is equilateral with side length 6 and  $D$  is the midpoint of  $BC$ . Determine the *exact* value of  $h$ , the height of  $\triangle ABC$ .



-  (b) In the diagram, a circle with centre  $O$  has radius 6. Regular hexagon  $EFGHIJ$  has sides of length 6 and vertices on the circle. Determine the *exact* area of the shaded region.







-  (c) A circle has centre  $O$  and radius  $r$ . A second circle has centre  $P$  and diameter  $MN$ . The circles intersect at  $M$  and  $N$ . If  $MN = r$ , determine the *exact* area of the shaded region, in terms of  $r$ .



4. The prime factorization of 45 is  $3^2 5^1$ . In general, the prime factorization of an integer  $n \geq 2$  is of the form  $p_1^{a_1} p_2^{a_2} p_3^{a_3} \dots p_k^{a_k}$  where  $p_1, p_2, \dots, p_k$  are different prime numbers and  $a_1, a_2, \dots, a_k$  are positive integers.

Given an input of an integer  $n \geq 2$ , the *Barbeau Process* outputs the number equal to  $n \left( \frac{a_1}{p_1} + \frac{a_2}{p_2} + \frac{a_3}{p_3} + \dots + \frac{a_k}{p_k} \right)$ .

For example, given an input of 45, the Barbeau Process outputs  $45 \left( \frac{2}{3} + \frac{1}{5} \right) = 30 + 9 = 39$ , since the prime factorization of 45 is  $3^2 5^1$ .

-  (a) Given an input of 126, what number does the Barbeau Process output?
-  (b) Determine all pairs  $(p, q)$  of different prime numbers such that the Barbeau Process with input  $p^2 q$  outputs 135.
-  (c) Determine all triples  $(a, b, c)$  of positive integers such that the Barbeau Process with input  $2^a 3^b 5^c$  outputs  $4 \times 2^a 3^b 5^c$ .
-  (d) Determine all integer values of  $n$  with  $2 \leq n < 10^{10}$  such that the Barbeau Process with input  $n$  outputs  $3n$ .



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Thank you for writing the 2016 Galois Contest! Each year, more than 220 000 students from more than 60 countries register to write the CEMC's Contests.

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