



The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING
cemc.uwaterloo.ca

2014 Pascal Contest

(Grade 9)

Thursday, February 20, 2014
(in North America and South America)

Friday, February 21, 2014
(outside of North America and South America)

Solutions

1. We evaluate the expression by first evaluating the expressions in brackets:

$$(8 \times 6) - (4 \div 2) = 48 - 2 = 46$$

ANSWER: (C)

2. Since the sum of the angles in a triangle is 180° , then $50^\circ + x^\circ + 45^\circ = 180^\circ$ or $x + 95 = 180$.
Therefore, $x = 85$.

ANSWER: (C)

3. 30% of 200 equals $\frac{30}{100} \times 200 = 60$.

Alternatively, we could note that 30% of 100 is 30 and $200 = 2 \times 100$, so 30% of 200 is 30×2 which equals 60.

ANSWER: (D)

4. Since $x = 3$, the side lengths of the figure are 4, 3, 6, and 10.

Thus, the perimeter of the figure is $4 + 3 + 6 + 10 = 23$.

(Alternatively, the perimeter is $x + 6 + 10 + (x + 1) = 2x + 17$. When $x = 3$, this equals $2(3) + 17$ or 23.)

ANSWER: (A)

5. The team earns 2 points for each win, so 9 wins earns $2 \times 9 = 18$ points.

The team earns 0 points for each loss, so 3 losses earns 0 points.

The team earns 1 point for each tie, so 4 ties earns 4 points.

In total, the team earns $18 + 0 + 4 = 22$ points.

ANSWER: (E)

6. The line representing a temperature of 3° is the horizontal line passing halfway between 2° and 4° on the vertical axis.

There are two data points on this line: one at 2 p.m. and one at 9 p.m.

The required time is 9 p.m.

ANSWER: (A)

7. *Solution 1*

We rewrite the left side of the given equation as $5 \times 6 \times (2 \times 3) \times (2 \times 3)$.

Since $5 \times 6 \times (2 \times 3) \times (2 \times 3) = 5 \times 6 \times n \times n$, then a possible value of n is 2×3 or 6.

Solution 2

Since $2 \times 2 \times 3 \times 3 \times 5 \times 6 = 5 \times 6 \times n \times n$, then $1080 = 30n^2$ or $n^2 = 36$.

Thus, a possible value of n is 6. (The second possible value for n is -6 .)

Solution 3

Dividing both sides by 5×6 , we obtain $2 \times 2 \times 3 \times 3 = n \times n$, which is equivalent to $n^2 = 36$.

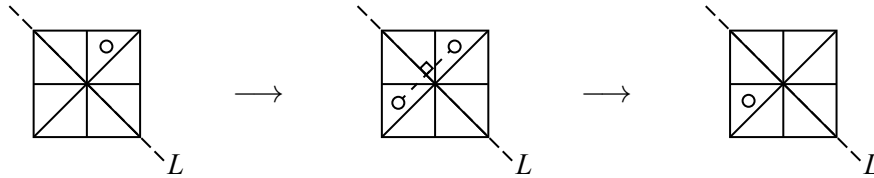
Thus, a possible value of n is 6. (The second possible value for n is -6 .)

ANSWER: (E)

8. *Solution 1*

The square, its 8 pieces and 2 diagonals are symmetrical about line L .

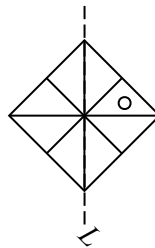
Upon reflection, the circle moves from the triangular above L which is adjacent to and above the second diagonal to the triangular region below L which is adjacent to and above the second diagonal.



Solution 2

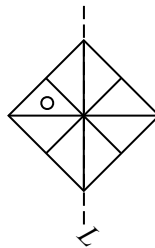
We can see that the final position of the figure is given in (D) by first rotating the original

figure by 45° clockwise to obtain



and then reflecting the figure through the

now vertical line to obtain



When we reposition the figure and line back to the

original position (by rotating counterclockwise by 45°), we obtain the figure in (D).

ANSWER: (D)

9. We note that $2^2 = 2 \times 2 = 4$, $2^3 = 2^2 \times 2 = 4 \times 2 = 8$, and $2^4 = 2^2 \times 2^2 = 4 \times 4 = 16$.
Therefore, $2^4 - 2^3 = 16 - 8 = 8 = 2^3$.

ANSWER: (D)

10. For $\frac{3}{4} + \frac{4}{\square} = 1$ to be true, we must have $\frac{4}{\square} = 1 - \frac{3}{4} = \frac{1}{4}$.

Since $\frac{1}{4} = \frac{4}{16}$, we rewrite the right side using the same numerator to obtain $\frac{4}{\square} = \frac{4}{16}$.

Therefore, $\square = 16$ makes the equation true.

(We can check that $\frac{3}{4} + \frac{4}{16} = 1$, as required.)

ANSWER: (E)

11. *Solution 1*

The faces not visible on the top cube are labelled with 2, 3 and 6 dots.

The faces not visible on the bottom cube are labelled with 1, 3, 4, and 5 dots.

Thus, the total number of dots on these other seven faces is $2 + 3 + 6 + 1 + 3 + 4 + 5 = 24$.

Solution 2

Since each of the two cubes has faces labelled with 1, 2, 3, 4, 5, and 6 dots, then the total number of dots on the two cubes is $2 \times (1 + 2 + 3 + 4 + 5 + 6) = 2 \times 21 = 42$.

The five visible faces have a total of $4 + 1 + 5 + 6 + 2 = 18$ dots.

Therefore, the seven other faces have a total of $42 - 18 = 24$ dots.

ANSWER: (E)

12. Since each \blacksquare has length $\frac{2}{3}$, then a strip of three \blacksquare will have length $3 \times \frac{2}{3} = 2$.
 We need two strips of length 2 to get a strip of length 4.
 Since 3 \blacksquare make a strip of length 2, then 6 \blacksquare make a strip of length 4.

ANSWER: (A)

13. We rearrange the given subtraction to create the addition statement $45 + 8Y = 1X2$.
 Next, we consider the units digits.
 From the statement, the sum $5 + Y$ has a units digit of 2. This means that $Y = 7$. This is the only possibility. Therefore, we have $45 + 87 = 1X2$.
 But $45 + 87 = 132$, so $X = 3$.
 Therefore, $X + Y = 3 + 7 = 10$.
 (We can check that $132 - 87 = 45$, as required.)

ANSWER: (C)

14. We simplify first, then substitute $x = 2y$:

$$(x + 2y) - (2x + y) = x + 2y - 2x - y = y - x = y - 2y = -y$$

Alternatively, we could substitute first, then simplify:

$$(x + 2y) - (2x + y) = (2y + 2y) - (2(2y) + y) = 4y - 5y = -y$$

ANSWER: (B)

15. *Solution 1*

Since $\triangle RPS$ is right-angled at P , then by the Pythagorean Theorem, $PR^2 + PS^2 = RS^2$ or $PR^2 + 18^2 = 30^2$.

This gives $PR^2 = 30^2 - 18^2 = 900 - 324 = 576$, from which $PR = 24$, since $PR > 0$.

Since P, S and Q lie on a straight line and RP is perpendicular to this line, then RP is actually a height for $\triangle QRS$ corresponding to base SQ .

Thus, the area of $\triangle QRS$ is $\frac{1}{2}(24)(14) = 168$.

Solution 2

Since $\triangle RPS$ is right-angled at P , then by the Pythagorean Theorem, $PR^2 + PS^2 = RS^2$ or $PR^2 + 18^2 = 30^2$.

This gives $PR^2 = 30^2 - 18^2 = 900 - 324 = 576$, from which $PR = 24$, since $PR > 0$.

The area of $\triangle QRS$ equals the area of $\triangle RPQ$ minus the area of $\triangle RPS$.

Since $\triangle RPQ$ is right-angled at P , its area is $\frac{1}{2}(PR)(PQ) = \frac{1}{2}(24)(18 + 14) = 12(32) = 384$.

Since $\triangle RPS$ is right-angled at P , its area is $\frac{1}{2}(PR)(PS) = \frac{1}{2}(24)(18) = 12(18) = 216$.

Therefore, the area of $\triangle QRS$ is $384 - 216 = 168$.

ANSWER: (B)

16. From the second row, $\triangle + \triangle + \triangle + \triangle = 24$ or $4\triangle = 24$, and so $\triangle = 6$.
 From the first row, $\heartsuit + \triangle + \triangle + \heartsuit = 26$ or $2\heartsuit + 2\triangle = 26$.
 Since $\triangle = 6$, then $2\heartsuit = 26 - 12 = 14$, and so $\heartsuit = 7$.
 From the fourth row, $\square + \heartsuit + \square + \triangle = 33$.
 Since $\triangle = 6$ and $\heartsuit = 7$, then $2\square + 7 + 6 = 33$, and so $2\square = 20$ or $\square = 10$.
 Finally, from the third row, $\square + \blacklozenge + \heartsuit + \blacklozenge = 27$.
 Since $\square = 10$ and $\heartsuit = 7$, then $2\blacklozenge = 27 - 10 - 7 = 10$.
 Thus, $\blacklozenge = 5$.

ANSWER: (A)

17. The cube has six identical square faces, each of which is 30 by 30.

Therefore, the surface area of the cube is $6(30^2) = 5400$.

The rectangular solid has two faces that are 20 by 30, two faces that are 20 by L , and two faces that are 30 by L .

Thus, the surface area of the rectangular solid is $2(20)(30) + 2(20L) + 2(30L) = 100L + 1200$. Since the surface areas of the two solids are equal, then $100L + 1200 = 5400$ or $100L = 4200$, and so $L = 42$.

ANSWER: (C)

18. The equality of the ratios $x : 4$ and $9 : y$ is equivalent to the equation $\frac{x}{4} = \frac{9}{y}$. (Note that x and y are both positive so we are not dividing by 0.)

This equation is equivalent to the equation $xy = 4(9) = 36$.

Thus, we want to determine the number of pairs of integers (x, y) for which $xy = 36$.

Since the positive divisors of 36 are 1, 2, 3, 4, 6, 9, 12, 18, 36, then the desired pairs are

$$(x, y) = (1, 36), (2, 18), (3, 12), (4, 9), (6, 6), (9, 4), (12, 3), (18, 2), (36, 1)$$

There are 9 such pairs.

ANSWER: (D)

19. We make a chart that lists the possible results for the first spin down the left side, the possible results for the second spin across the top, and the product of the two results in the corresponding cells:

	1	2	3	4
1	1	2	3	4
2	2	4	6	8
3	3	6	9	12
4	4	8	12	16

Since each spin is equally likely to stop on 1, 2, 3, or 4, then each of the 16 products shown in the chart is equally likely.

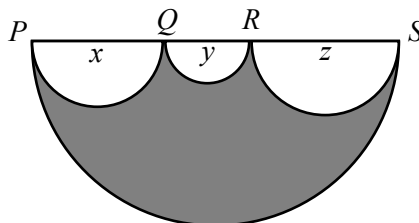
Since the product 4 appears three times in the table and this is more than any of the other numbers, then it is the product that is most likely to occur.

ANSWER: (B)

20. The perimeter of the shaded region consists of four pieces: a semi-circle with diameter PS , a semi-circle with diameter PQ , a semi-circle with diameter QR , and a semi-circle with diameter RS .

We note that since a circle with diameter d has circumference equal to πd , then, not including the diameter itself, the length of a semi-circle with diameter d is $\frac{1}{2}\pi d$.

Suppose that the diameter of semi-circle PQ is x , the diameter of semi-circle QR is y , and the diameter of semi-circle RS is z .



We are given that the diameter of the semi-circle PS is 4.

Since $PQ + QR + RS = PS$, then $x + y + z = 4$.

Thus, the perimeter of the shaded region is

$$\frac{1}{2}\pi(PQ) + \frac{1}{2}\pi(QR) + \frac{1}{2}\pi(RS) + \frac{1}{2}\pi(PS) = \frac{1}{2}\pi x + \frac{1}{2}\pi y + \frac{1}{2}\pi z + \frac{1}{2}\pi(4) = \frac{1}{2}\pi(x + y + z + 4) = \frac{1}{2}\pi(4 + 4) = 4\pi$$

We want to determine the area of the square whose perimeter equals this perimeter (that is, whose perimeter is 4π).

If a square has perimeter 4π , then its side length is $\frac{1}{4}(4\pi) = \pi$, and so its area is π^2 .

Because this problem is multiple choice, then we should get the same answer regardless of the actual lengths of PQ , QR and RS , since we are not told what these lengths are. Therefore, we can assign to these lengths arbitrary values that satisfy the condition $PQ + QR + RS = 4$. For example, if $PQ = QR = 1$ and $RS = 2$, then we can calculate the perimeter of the shaded region to be

$$\frac{1}{2}\pi(PQ) + \frac{1}{2}\pi(QR) + \frac{1}{2}\pi(RS) + \frac{1}{2}\pi(PS) = \frac{1}{2}\pi(1) + \frac{1}{2}\pi(1) + \frac{1}{2}\pi(2) + \frac{1}{2}\pi(4) = \frac{1}{2}\pi(1 + 1 + 2 + 4) = 4\pi$$

and obtain the same answer as above.

ANSWER: (C)

21. *Solution 1*

The ratio of side lengths in the given 4×6 grid is $4 : 6$ which is equivalent to $2 : 3$.

The ratio of side lengths in the desired 30×45 grid is $30 : 45$ which is also equivalent to $2 : 3$.

Therefore, the 30×45 grid can be built using 2×3 blocks; the resulting grid can be seen as a 15×15 array of 2×3 blocks.

The diagonal line of the 30×45 grid has slope $\frac{30}{45} = \frac{2}{3}$ and so passes through the lower left and upper right corners of each of the diagonal blocks of the 15×15 array of 2×3 blocks. Each of these corners is a lattice point. The diagonal line does not pass through any other lattice point within each of these diagonal blocks, as can be seen in the 4×6 grid.

We note that the upper right corner of a diagonal block is the same point as the lower left corner of the next such block. This means that we have to be careful with our counting.

There are 15 diagonal blocks. The diagonal line passes through the bottom left corner of the grid and passes through the upper right corner of each of the diagonal blocks.

This means that the diagonal line passes through $1 + 15 = 16$ lattice points.

Solution 2

We assign coordinates to the desired 30×45 grid, with the bottom left corner at the origin $(0, 0)$, the vertical side lying along the positive y -axis from $(0, 0)$ to $(0, 30)$ and the horizontal side lying along the positive x -axis from $(0, 0)$ to $(45, 0)$.

The upper right corner of the grid has coordinates $(45, 30)$. The grid lines are the horizontal lines $y = 0, y = 1, \dots, y = 29, y = 30$ and the vertical lines $x = 0, x = 1, \dots, x = 44, x = 45$.

The lattice points in the grid are the points with integer coordinates.

Consider the diagonal that joins $(0, 0)$ to $(45, 30)$.

The slope of this line is $\frac{30-0}{45-0} = \frac{2}{3}$.

Since the diagonal passes through the origin, its equation is $y = \frac{2}{3}x$.

Thus, we must determine the number of lattice points that lie on the line $y = \frac{2}{3}x$ with x -coordinates between $x = 0$ and $x = 45$, inclusive.

Suppose that (a, b) is a lattice point on the line; that is, a and b are both integers.

Since $b = \frac{2}{3}a$, then for b to be an integer, it must be the case that a is a multiple of 3.

The multiples of 3 between 0 and 45, inclusive, are $0, 3, 6, \dots, 42, 45$.

Since $45 = 15(3)$, then there are 16 numbers in the list.

Each of these values for a gives an integer for b and so gives a lattice point on the line.

Thus, there are 16 lattice points on the diagonal.

ANSWER: (B)

22. Since there are four triangular sections in each flag, then at most four colours can be used in a single flag.

Since no two adjacent triangles are the same colour, then at least two colours must be used. (For example, the sections Top and Left must be different colours.)

Therefore, the number of colours used is 2, 3 or 4.

We count the number of possible flags in each case.

Case 1: 2 colours

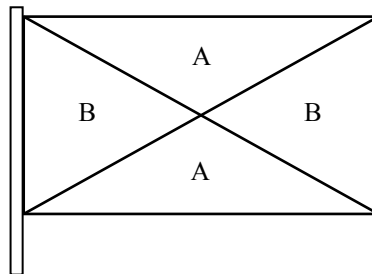
We call the colours A and B.

Assign the colour A to Top.

Since Left and Right cannot be coloured A and there is only one other colour, then Left and Right are both coloured B.

Bottom cannot be coloured B (since it shares an edge with Left and Right) so must be coloured A.

This gives us:



This configuration does not violate the given rule.

There are 5 possible colours for A (red, white, blue, green, purple).

For each of these 5 choices, there are 4 possible colours for B (any of the remaining 4 colours).

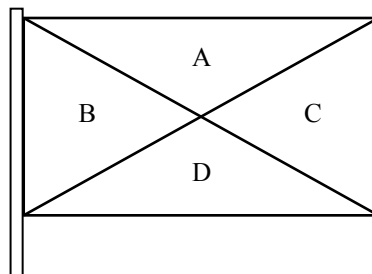
Therefore, there are $5(4) = 20$ possible flags in this case.

Case 2: 4 colours

We call the colours A, B, C, and D.

Since there are 4 sections and 4 colours used, then each section is a different colour.

We label them as shown:



This configuration does not violate the given rule.

There are 5 possible colours for A. For each of these 5 choices, there are 4 possible colours for B. For each of these combinations, there are 3 possible colours for C and 2 possible colours for D.

Therefore, there are $5(4)(3)(2) = 120$ possible flags in this case.

Case 3: 3 colours

We call the colours A, B and C.

Assign the colour A to Top.

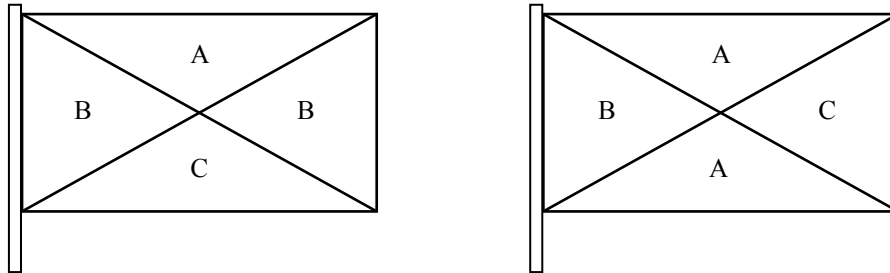
Since Left cannot be coloured A, we assign it the colour B.

Section Right cannot be coloured A, so could be B or C.

If Right is coloured B, then in order to use all three colours, Bottom must be coloured C.

If Right is coloured C, then Bottom (which shares an edge with each of Left and Right) must be coloured A.

This gives two possible configurations:



Neither configuration violates the given rule.

In each configuration, there are 5 possible colours for A. For each of these 5 choices, there are 4 possible colours for B. For each of these combinations, there are 3 possible colours for C.

Since there are two such configurations, then there are $2(5)(4)(3) = 120$ possible flags in this case.

In total, there are $20 + 120 + 120 = 260$ possible flags.

ANSWER: (E)

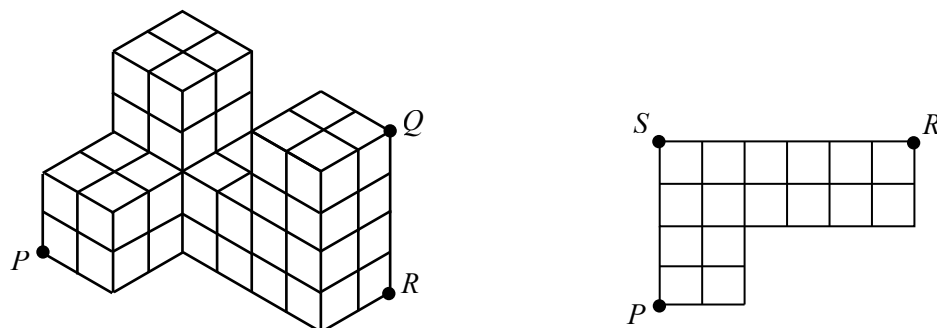
23. First, we calculate the distance PQ in terms of n .

Suppose that R is the point at the bottom of the solid directly under Q and S is the back left bottom corner of the figure (unseen in the problem's diagram).

Since QR is perpendicular to the bottom surface of the solid, then $\triangle PRQ$ is right-angled at R and so $PQ^2 = PR^2 + RQ^2$.

We note also that $\triangle PSR$ is right-angled at S , since the solid is made up of cubes.

Therefore, $PR^2 = PS^2 + SR^2$.



This tells us that $PQ^2 = PS^2 + SR^2 + RQ^2$.

Note that the distance PS equals four times the edge length of one of the small cubes, SR is six times the edge length, and RQ is four times the edge length.

Since the edge length is \sqrt{n} , then $PS = 4\sqrt{n}$, $SR = 6\sqrt{n}$, and $RQ = 4\sqrt{n}$.

Thus, $PQ^2 = (4\sqrt{n})^2 + (6\sqrt{n})^2 + (4\sqrt{n})^2 = 16n + 36n + 16n = 68n$.

Therefore, $PQ = \sqrt{68n}$.

We need to determine the smallest positive integer n for which $\sqrt{68n}$ is an integer.

For $\sqrt{68n}$ to be an integer, $68n$ must be a perfect square.

Note that $68n = 4(17n) = 2(2)(17)(n)$.

A positive integer is a perfect square whenever each prime factor occurs an even number of times.

Thus, for $68n$ to be a perfect square, n must include a factor of 17.

The smallest possible such n is in fact $n = 17$.

When $n = 17$, we have $68n = 2(2)(17)(17) = (2 \times 17)^2$ which is a perfect square.

Therefore, the smallest positive integer n for which the distance PQ is an integer is $n = 17$.

(Once we determined that $PQ = \sqrt{68n}$, we could have tried the five possible answers from smallest to largest until we obtained an integer value for PQ .)

ANSWER: (A)

24. As Nadia walks from N to G , suppose that she walks x km uphill and y km downhill. We are told that she walks 2.5 km on flat ground.

This means that when she walks from G to N , she will walk x km downhill, y km uphill, and again 2.5 km on flat ground. This is because downhill portions become uphill portions on the return trip, while uphill portions become downhill portions on the return trip.

We are told that Nadia walks at 5 km/h on flat ground, 4 km/h uphill, and 6 km/h downhill.

Since $\text{speed} = \frac{\text{distance}}{\text{time}}$, then $\text{distance} = \text{speed} \times \text{time}$ and $\text{time} = \frac{\text{distance}}{\text{speed}}$.

Thus, on her trip from N to G , her time walking uphill is $\frac{x}{4}$ hours, her time walking downhill is $\frac{y}{6}$ hours, and her time walking on flat ground is $\frac{2.5}{5}$ hours.

Since it takes her 1 hour and 36 minutes (which is 96 minutes or $\frac{96}{60}$ hours), then

$$\frac{x}{4} + \frac{y}{6} + \frac{2.5}{5} = \frac{96}{60}$$

A similar analysis of the return trip gives

$$\frac{x}{6} + \frac{y}{4} + \frac{2.5}{5} = \frac{99}{60}$$

We are asked for the total distance from N to G , which equals $x + y + 2.5$ km. Therefore, we need to determine $x + y$.

We add the two equations above and simplify to obtain

$$\begin{aligned} \frac{x}{4} + \frac{x}{6} + \frac{y}{6} + \frac{y}{4} + 1 &= \frac{195}{60} \\ x \left(\frac{1}{4} + \frac{1}{6} \right) + y \left(\frac{1}{4} + \frac{1}{6} \right) &= \frac{135}{60} \\ \frac{5}{12}x + \frac{5}{12}y &= \frac{9}{4} \\ x + y &= \frac{12}{5} \left(\frac{9}{4} \right) \end{aligned}$$

Thus, $x + y = \frac{108}{20} = \frac{27}{5} = 5.4$ km.

Finally, the distance from N to G is $5.4 + 2.5 = 7.9$ km.

ANSWER: (E)

25. First, we simplify $\frac{2009}{2014} + \frac{2019}{n}$ to obtain $\frac{2009n + 2014(2019)}{2014n}$ or $\frac{2009n + 4\,066\,266}{2014n}$.

Since $\frac{2009n + 4\,066\,266}{2014n} = \frac{a}{b}$ and $\frac{a}{b}$ is in lowest terms, then $2009n + 4\,066\,266 = ka$

and $2014n = kb$ for some positive integer k .

Since $2009n + 4\,066\,266 = ka$, then if a is a multiple of 1004, we must have that $2009n + 4\,066\,266$ is a multiple of 1004 as well.

Therefore, we determine the values of n for which $2009n + 4\,066\,266$ is divisible by 1004 and from this list find the smallest such n that makes a divisible by 1004. (Note that even if $2009n + 4\,066\,266$ is divisible by 1004, it might not be the case that a is divisible by 1004, since reducing the fraction $\frac{2009n + 4\,066\,266}{2014n}$ might eliminate some or all of the prime factors of 1004 in the numerator.)

We note that $2008 = 2 \times 1004$ and $4\,066\,200 = 4050 \times 1004$, so we write

$$2009n + 4\,066\,266 = (2008n + 4\,066\,200) + (n + 66) = 1004(2n + 4050) + (n + 66)$$

(2008 and 4 066 200 are the largest multiples of 1004 less than 2009 and 4 066 266, respectively.)

Since $1004(2n + 4050)$ is a multiple of 1004, then $2009n + 4\,066\,266$ is a multiple of 1004 whenever $n + 66$ is a multiple of 1004, say $1004m$ (that is, $n + 66 = 1004m$).

Thus, $2009n + 4\,066\,266$ is a multiple of 1004 whenever $n = 1004m - 66$ for some positive integer m .

These are the values of n for which the expression $2009n + 4\,066\,266$ is divisible by 1004. Now we need to determine the smallest of these n for which a is divisible by 1004.

When $m = 1$, we have $n = 1004 - 66 = 938$. In this case,

$$\frac{2009n + 4\,066\,266}{2014n} = \frac{2009(938) + 4\,066\,2006}{2014(938)} = \frac{5\,950\,708}{1\,889\,132} = \frac{1\,487\,677}{472\,283}$$

where in the last step we divided a common factor of 4 out of the numerator and denominator. Regardless of whether this last fraction is in lowest terms, its numerator is odd and so $\frac{a}{b}$ (the equivalent lowest terms fraction) will also have a odd, so a cannot be divisible by 1004. So, we try the next value of m .

When $m = 2$, we have $n = 2008 - 66 = 1942$. In this case,

$$\frac{2009n + 4\,066\,266}{2014n} = \frac{2009(1942) + 4\,066\,2006}{2014(1942)} = \frac{7\,967\,744}{3\,911\,188} = \frac{1\,991\,936}{977\,797}$$

where in the last step we divided a common factor of 4 out of the numerator and denominator. Now $1004 = 4 \times 251$ and 251 is a prime number. (251 is a prime number because it is not divisible by any of the primes 2, 3, 5, 7, 11, and 13, which are all of the primes less than $\sqrt{251}$.) Now $1\,991\,936 = 1984 \times 1004$ so is a multiple of 1004, and 977 797 is not divisible by 4 or by 251. This means that when $\frac{1\,991\,936}{977\,797}$ is written in lowest terms as $\frac{a}{b}$, then a will be divisible by 1004.

Therefore, the smallest value of n with the desired property is $n = 1942$, which has a sum of digits equal to $1 + 9 + 4 + 2 = 16$.

ANSWER: (A)