The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING
cemc.uwaterloo.ca

2014 Gauss Contests

(Grades 7 and 8)

Wednesday, May 14, 2014
(in North America and South America)

Thursday, May 15, 2014
(outside of North America and South America)

Solutions

Grade 7

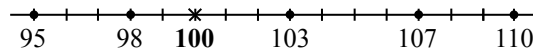1. Evaluating, $(4 \times 3) + 2 = 12 + 2 = 14$.

   ANSWER: (C)

2. *Solution 1*

   We place each of the five answers and 100 on a number line.

   Of the five answers given, the two closest numbers to 100 are 98 and 103.

   Since 98 is 2 units away from 100 and 103 is 3 units away from 100, then 98 is closest to 100.

   Solution 2

   We calculate the positive difference between 100 and each of the five possible answers.

   The number closest to 100 on the number line will produce the smallest of these positive differences.

Possible Answers	98	95	103	107	110
Positive Differences	$100 - 98 = 2$	$100 - 95 = 5$	$103 - 100 = 3$	$107 - 100 = 7$	$110 - 100 = 10$

   Since the smallest positive difference is 2, then 98 is the closest to 100 on the number line.

   ANSWER: (A)

3. Since five times the number equals one hundred, then the number equals one hundred divided by five. Therefore the number is $100 \div 5 = 20$.

   ANSWER: (E)

4. The spinner has 6 sections in total and 2 of these sections contain the letter Q.

   Sections are equal to one another in size and thus they are each equally likely to be landed on.

   Therefore, the probability of landing on a section that contains the letter Q is $\frac{2}{6}$.

   ANSWER: (D)

5. Each scoop of fish food can feed 8 goldfish.

   Therefore, 4 scoops of fish food can feed $4 \times 8 = 32$ goldfish.

   ANSWER: (E)

6. Both the numerator and the denominator are divisible by 5.

   Dividing, we get $\frac{15}{25} = \frac{15 \div 5}{25 \div 5} = \frac{3}{5}$. Therefore, $\frac{15}{25}$ is equivalent to $\frac{3}{5}$.

   ANSWER: (C)

7. The largest two-digit number that is a multiple of 7 is $7 \times 14 = 98$.

   Thus, there are 14 positive multiples of 7 that are less than 100.

   However, this includes $7 \times 1 = 7$ which is not a two-digit number.

   Therefore, there are $14 - 1 = 13$ positive two-digit numbers which are divisible by 7.

   (Note that these 13 numbers are $14, 21, 28, 35, 42, 49, 56, 63, 70, 77, 84, 91$, and 98.)

   ANSWER: (E)

8. *Solution 1*

   Evaluating the left side of the equation, we get $9210 - 9124 = 86$.

   Therefore the right side of the equation, $210 - \square$, must also equal 86.

   Since $210 - 124 = 86$, then the value represented by the $\square$ is 124.

   Solution 2

   Since $9210 - 9124 = (9000 + 210) - (9000 + 124) = 9000 - 9000 + 210 - 124 = 210 - 124$, then the value represented by the $\square$ is 124.
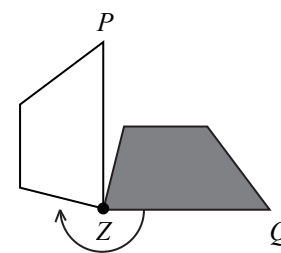
   ANSWER: (D)

9. The measure of $\angle PZQ$ formed by the bottom edge of the shaded quadrilateral, ZQ, and this same edge after the rotation, ZP, is approximately 90°.

   The transformation of the shaded quadrilateral to the unshaded quadrilateral is a clockwise rotation around point Z through an angle equal to the measure of reflex angle PZQ.

   The measure of $\angle PZQ$ added to the measure of reflex $\angle PZQ$ is equal to the measure of one complete rotation, or 360°.

   Therefore, the measure of reflex angle PZQ is approximately $360° - 90°$ or 270°.

   Thus the clockwise rotation around point Z is through an angle of approximately 270°.

   ANSWER: (B)

10. In the table below, each of the five expressions is evaluated using the correct order of operations.

Expression	Value
(A) $3 - 4 \times 5 + 6$	$3 - 20 + 6 = -17 + 6 = -11$
(B) $3 \times 4 + 5 \div 6$	$12 + 5 \div 6 = 12 + \frac{5}{6} = 12\frac{5}{6}$
(C) $3 + 4 \times 5 - 6$	$3 + 20 - 6 = 23 - 6 = 17$
(D) $3 \div 4 + 5 - 6$	$\frac{3}{4} + 5 - 6 = 5\frac{3}{4} - 6 = \frac{23}{4} - \frac{24}{4} = -\frac{1}{4}$
(E) $3 \times 4 \div 5 + 6$	$12 \div 5 + 6 = \frac{12}{5} + 6 = 2\frac{2}{5} + 6 = 8\frac{2}{5}$

   The only expression that is equal to 17 is $3 + 4 \times 5 - 6$, or (C).

   ANSWER: (C)

11. Since each of the numbers in the set is between 0 and 1, then the tenths digit of each number contributes more to its value than any of its other digits.

   The largest tenths digit of the given numbers is 4, and so 0.43 is the largest number in the set. The smallest tenths digit of the given numbers is 0, so 0.034 is the smallest number in the set. Therefore, the sum of the smallest number in the set and the largest number in the set is $0.034 + 0.43 = 0.464$.

   ANSWER: (D)

12. The two diagonals of a square bisect one another (divide each other into two equal lengths) at the centre of the square.

   Therefore, the two diagonals divide the square into four identical triangles.

   One of these four triangles is the shaded region which has area equal to one quarter of the area of the square.

   Since the area of the square is $8 \times 8 = 64$ cm^2, the area of the shaded region is $64 \div 4 = 16$ cm^2.

   ANSWER: (C)

13. The sum of the three numbers in the first column is $13 + 14 + 9 = 36$.

   The sum of the numbers in each column and in each row in the square is the same and so the sum of the three numbers in the second row is also 36.

   That is, $14 + x + 10 = 36$ or $x + 24 = 36$, and so $x = 36 - 24 = 12$.

   ANSWER: (E)

14. We systematically count rectangles by searching for groups of rectangles that are of similar size. The largest rectangles in the diagram are all roughly the same size and overlap in pairs. There are 3 of these; each is shaded black and shown below.
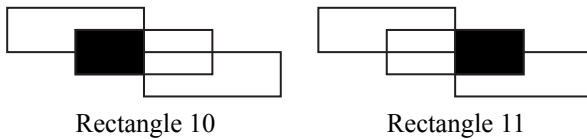

Rectangle 1   Rectangle 2   Rectangle 3
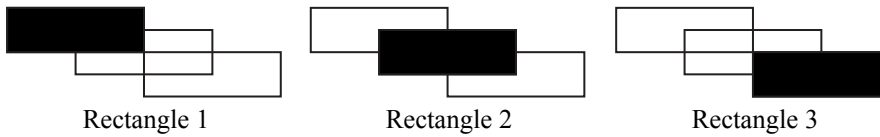
Rectangle 2 (shown above) consists of 4 small rectangles.
We shade these rectangles black and label them $4, 5, 6, 7$, as shown below.

Rectangle 4   Rectangle 5   Rectangle 6   Rectangle 7
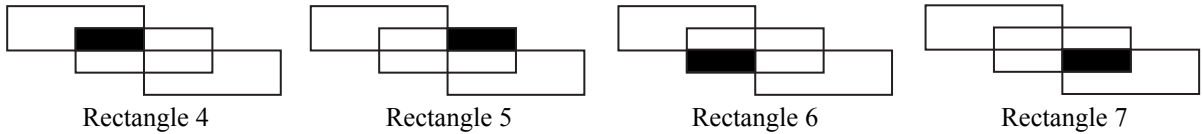
Together, Rectangle 4 and Rectangle 5 (shown above) create Rectangle 8, shown below.
Similarly, Rectangle 6 and Rectangle 7 together create Rectangle 9, shown below.

Rectangle 8   Rectangle 9
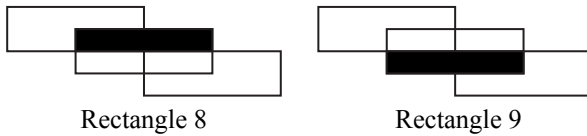
Finally, Rectangle 4 and Rectangle 6 (shown above) together create Rectangle 10, shown below.
Similarly, Rectangle 5 and Rectangle 7 together create Rectangle 11, shown below.

Rectangle 10   Rectangle 11

There are no other rectangles that can be formed.
In total, there are 11 rectangles in the given diagram.

ANSWER: (A)

15. The horizontal translation needed to get from Lori's house to Alex's house is the difference between the x-coordinate of Lori's house, 6, and the x-coordinate of Alex's house, -2, or $6 - (-2) = 6 + 2 = 8$.
The vertical translation needed to get from Lori's house to Alex's house is the difference between the y-coordinate of Lori's house, 3, and the y-coordinate of Alex's house, -4, or $3 - (-4) = 3 + 4 = 7$.
From Lori's house, Alex's house is left and down.
Therefore the translation needed to get from Lori's house to Alex's house is 8 units left and 7 units down.
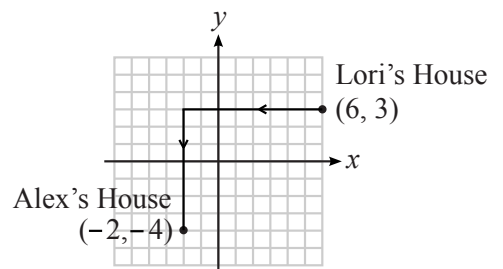
ANSWER: (D)

16. Reading from the graph, Riley-Ann scored 8 points in Game 1, 7 points in Game 2, 20 points in Game 3, 7 points in Game 4, and 18 points in Game 5.
Therefore the mean number of points that she scored per game is $\frac{8+7+20+7+18}{5} = \frac{60}{5} = 12$.
Since the ordered list (smallest to largest) of the number of points scored per game is $7, 7, 8, 18, 20$, then the median is 8, the number in the middle of this ordered list.
The difference between the mean and the median of the number of points that Riley-Ann scored is $12 - 8 = 4$.

ANSWER: (D)

17. *Solution 1*
    Since PQR is a straight line segment, then $\angle PQR = 180°$.
    Since $\angle SQP + \angle SQR = 180°$, then $\angle SQR = 180° - \angle SQP = 180° - 75° = 105°$.
    The three angles in a triangle add to 180°, so $\angle QSR + \angle SQR + \angle QRS = 180°$, or
    $\angle QSR = 180° - \angle SQR - \angle QRS = 180° - 105° - 30° = 45°$.

    Solution 2
    The exterior angle of a triangle is equal to the sum of the two non-adjacent interior angles of
    the triangle.
    Since $\angle SQP$ is an exterior angle of $\triangle SQR$, and the two opposite interior angles are $\angle QSR$
    and $\angle QRS$, then $\angle SQP = \angle QSR + \angle QRS$.
    Thus, $75° = \angle QSR + 30°$ or $\angle QSR = 75° - 30° = 45°$.

    Answer: (E)

18. *Solution 1*
    The outer square has an area of 9 cm^2, so the sides of this outer
    square have length 3 cm (since $3 \times 3 = 9$), and thus $PN = 3$ cm.
    The inner square has an area of 1 cm^2, so the sides of this inner
    square have length 1 cm (since $1 \times 1 = 1$), and thus $MR = 1$ cm.
    Since $PN = 3$ cm, then $PS + SN = 3$ cm and so $QR + SN = 3$ cm
    (since $QR = PS$).
    But $QR = QM + MR$, so then $QM + MR + SN = 3$ cm or
    $QM + 1 + SN = 3$ cm (since $MR = 1$ cm).
    From this last equation we get $QM + SN = 2$ cm.
    Since each of QM and SN is the width of an identical rectangle, then $QM = SN = 1$ cm.
    Using $PS + SN = 3$ cm, we get $PS + 1 = 3$ cm and so $PS = 2$ cm.
    Since the rectangles are identical, then $SN = PQ = 1$ cm.
    The perimeter of rectangle $PQRS$ is $2 \times (PS + PQ) = 2 \times (2 + 1) = 2 \times 3 = 6$ cm.
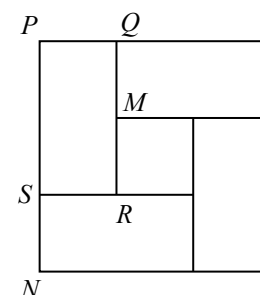
    Solution 2
    The outer square has an area of 9 cm^2, so the sides of this outer
    square have length 3 cm (since $3 \times 3 = 9$), and thus $PN = 3$ cm.
    Since $PN = 3$ cm, then $PS + SN = 3$ cm.
    Since each of PQ and SN is the width of an identical rectangle, then
    $PQ = SN$ and so $PS + SN = PS + PQ = 3$ cm.
    The perimeter of $PQRS$ is $2 \times (PS + PQ) = 2 \times 3 = 6$ cm.

    Answer: (A)

19. The width of Sarah's rectangular floor is 18 hand lengths.
    Since each hand length is 20 cm, then the width of the floor is $18 \times 20 = 360$ cm.
    The length of Sarah's rectangular floor is 22 hand lengths.
    Since each hand length is 20 cm, then the length of the floor is $22 \times 20 = 440$ cm.
    Thus the area of the floor is $360 \times 440 = 158\,400$ cm^2.
    Of the given answers, the closest to the area of the floor is $160\,000$ cm^2.

    Answer: (A)

20. *Solution 1*
    Since $20 \times 20 \times 20 = 8000$ and $30 \times 30 \times 30 = 27\,000$, then we might guess that the three
    consecutive odd numbers whose product is 9177 are closer to 20 than they are to 30.
    Using trial and error, we determine that $21 \times 23 \times 25 = 12\,075$, which is too large.

The next smallest set of three consecutive odd numbers is $19, 21, 23$ and the product of these three numbers is $19 \times 21 \times 23 = 9177$, as required.

Thus, the sum of the three consecutive odd numbers whose product is 9177 is $19+21+23 = 63$.

Solution 2

We begin by determining the prime numbers whose product is 9177. (This is called the *prime factorization* of 9177.)

This prime factorization of 9177 is shown in the factor tree to the right. That is, $9177 = 3 \times 3059 = 3 \times 7 \times 437 = 3 \times 7 \times 19 \times 23$. Since $3 \times 7 = 21$, then $9177 = 21 \times 19 \times 23$ and so the three consecutive numbers whose product is 9177 are $19, 21, 23$.

Thus, the sum of the three consecutive odd numbers whose product is 9177 is $19 + 21 + 23 = 63$.
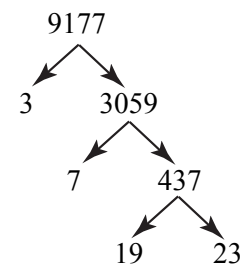
ANSWER: (D)

21. At Store Q, the bicycle's regular price is 15% more than the price at Store P, or 15% more than $200.

Since 15% of 200 is $\frac{15}{100} \times 200 = 0.15 \times 200 = 30$, then 15% more than $200 is $200+$30 or $230.

This bicycle is on sale at Store Q for 10% off of the regular price, $230.

Since 10% of 230 is $\frac{10}{100} \times 230 = 0.10 \times 230 = 23$, then 10% off of $230 is $230 - $23 or $207.

The sale price of the bicycle at Store Q is $207.

ANSWER: (D)

22. Assume the top face of the cube is coloured green.

Since the front face of the cube shares an edge with the top face, it cannot be coloured green. Thus, we need at least two colours.

Thus, we assume that the front face is coloured blue, as shown in Figure 1.

Since the right face shares an edge with the top face and with the front face, it cannot be coloured green or blue. Thus, we need at least three colours.

Thus, we assume that the right face is coloured red, as shown in Figure 2.

We have shown that at least 3 colours are needed. In fact, the cube can be coloured with exactly 3 colours by colouring the left face red, the back face blue, and the bottom face green (Figure 3).

In this way, the cube is coloured with exactly 3 colours and no two faces that share an edge are the same colour.

Therefore, 3 is the smallest number of colours needed to paint a cube so that no two faces that share an edge are the same colour.
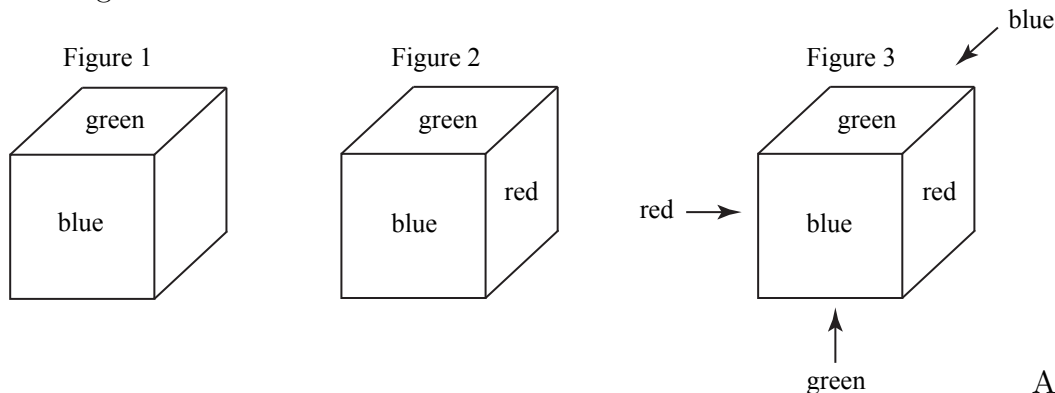
ANSWER: (B)

23. *Solution 1*

For each of the 6 possible outcomes that could appear on the red die, there are 6 possible outcomes that could appear on the blue die.

That is, the total number of possible outcomes when a standard six-sided red die and a standard six-sided blue die are rolled is $6 \times 6 = 36$.

These 36 outcomes are shown in the table below.

When a number that appears on the red die is greater than a number that appears on the blue die, a checkmark has been placed in the appropriate cell, corresponding to the intersection of the column and row.

For example, the table cell containing the double checkmark ✓✓ represents the outcome of a 4 appearing on the red die and a 2 appearing on the blue die.

Number on the Red Die

Red / Blue	1	2	3	4	5	6
1		✓	✓	✓	✓	✓
2			✓	✓✓	✓	✓
3				✓	✓	✓
4					✓	✓
5						✓
6						

(Number on the Blue Die)

Of the 36 possible outcomes, $1 + 2 + 3 + 4 + 5$ or 15 have a number appearing on the red die that is larger than the number appearing on the blue die.

The probability that the number appearing on the red die is greater than the number appearing on the blue die is $\frac{15}{36}$.

Solution 2

As in Solution 1, we determine the total number of possible outcomes to be 36.

Each of these 36 outcomes can be grouped into one of three possibilities; the number appearing on the red die is greater than the number appearing on the blue die, the number appearing on the red die is less than the number appearing on the blue die, or the numbers appearing on the two dice are equal.

There are 6 possible outcomes in which the numbers appearing on the two dice are equal (both numbers are 1, both numbers are 2, and so on).

Of the 36 total outcomes, this leaves $36 - 6 = 30$ outcomes in which either the number appearing on the red die is greater than the number appearing on the blue die, or the number appearing on the red die is less than the number appearing on the blue die.

These two possibilities are equally likely to happen (since both dice are identical except for colour), and so the number appearing on the red die will be greater than the number appearing on the blue die in half of the 30 outcomes, or 15 outcomes.

Thus, the probability that the number appearing on the red die is greater than the number appearing on the blue die is $\frac{15}{36}$.

ANSWER: (C)

24. We begin by joining Q to P.
    Since Q and P are the midpoints of ST and UV, then QP is parallel to both SV and TU and rectangles $SQPV$ and $QTUP$ are identical.
    In rectangle $SQPV$, VQ is a diagonal.
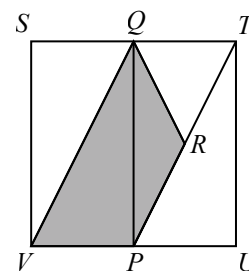   Similarly, since PR is parallel to VQ then PR extended to T is a diagonal of rectangle $QTUP$, as shown in Figure 1.

    Figure 1

    In Figure 2, we label points A, B, C, D, E, and F, the midpoints of SQ, QT, TU, UP, PV, and VS, respectively.
    We join A to E, B to D and F to C, with FC intersecting QP at the centre of the square O, as shown.
    Since $PR = QR$ and R lies on diagonal PT, then both FC and BD pass through R. (That is, R is the centre of $QTUP$.)
    The line segments AE, QP, BD, and FC divide square $STUV$ into 8 identical rectangles.
    In one of these rectangles, $QBRO$, diagonal QR divides the rectangle into 2 equal areas.
    That is, the area of $\triangle QOR$ is half of the area of rectangle $QBRO$.
    Similarly, the area of $\triangle POR$ is half of the area of rectangle $PORD$.

    Figure 2

    Rectangle $SQPV$ has area equal to 4 of the 8 identical rectangles.
    Therefore, $\triangle QPV$ has area equal to 2 of the 8 identical rectangles (since diagonal VQ divides the area of $SQPV$ in half).
    Thus the total shaded area, which is $\triangle QOR + \triangle POR + \triangle QPV$, is equivalent to the area of $\frac{1}{2} + \frac{1}{2} + 2$ or 3 of the identical rectangles.
    Since square $STUV$ is divided into 8 of these identical rectangles, and the shaded area is equivalent to the area of 3 of these 8 rectangles, then the unshaded area occupies an area equal to that of the remaining $8 - 3$ or 5 rectangles.
    Therefore, the ratio of the shaded area to the unshaded area is $3 : 5$.
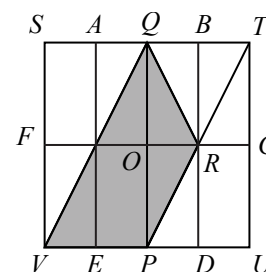
    ANSWER: (B)

25. We begin by listing the first few line segment endpoints as determined by the number of segments that Paul has drawn.

Line Segment Number	Endpoint of the Line Segment
1	$(1, 0)$
2	$(1, 2)$
3	$(4, 2)$
4	$(4, 6)$
5	$(9, 6)$
6	$(9, 12)$
7	$(16, 12)$
8	$(16, 20)$

    Since each new line segment is drawn either vertically or horizontally from the endpoint of the previous line segment, then only one of the x-coordinate or y-coordinate changes from one endpoint to the next endpoint.
    Further, since the odd numbered line segments are horizontal, the x-coordinate of the endpoint of these segments changes from the previous endpoint while the y-coordinate remains the same.

Similarly, since the even numbered line segments are vertical, the y-coordinate of the endpoint of these segments changes from the previous endpoint while the x-coordinate remains the same.

We are given that one of the line segments ends at the point $(529, 506)$.
We will begin by determining the number of segments that must be drawn for the x-coordinate of the endpoint of the final line segment to be 529.
The first line segment is drawn horizontally from the origin, to the right, has length 1, and thus ends with an x-coordinate of 1.
The third line segment is drawn horizontally to the right, has length 3 and thus ends with an x-coordinate of $1 + 3 = 4$.
The fifth line segment is drawn horizontally to the right, has length 5 and thus ends with an x-coordinate of $1 + 3 + 5 = 9$.
To get an idea of how many line segments are required before the x-coordinate of the endpoint is 529, we begin by considering the first 21 line segments.
The endpoint of the 21st line segment has x-coordinate equal to $1 + 3 + 5 + \cdots + 17 + 19 + 21$.
To make this sum easier to determine, we rearrange the terms to get
$(1+21)+(3+19)+(5+17)+(7+15)+(9+13)+11 = 22+22+22+22+22+11 = 22 \times 5 + 11 = 121$.
Since 121 is much smaller than 529, we continue to try larger numbers of line segments until we finally reach 45 line segments.
The endpoint of the 45th line segment has x-coordinate equal to $1 + 3 + 5 + \cdots + 41 + 43 + 45$.
To make this sum easier to determine, we rearrange the terms to get
$$(1 + 45) + (3 + 43) + (5 + 41) + \cdots + (19 + 27) + (21 + 25) + 23 = 46 \times 11 + 23 = 529.$$
Therefore, the 45th line segment ends with an x-coordinate of 529.
The next line segment (the 46th) will also have an endpoint with this same x-coordinate, 529, since even numbered line segments are vertical (thus, only the y-coordinate will change).
This tells us that $(529, 506)$ is the endpoint of either the 45th or the 46th line segment.

At this point we may confirm that 506 is the y-coordinate of the endpoint of the 45th line segment (and hence the 44th line segment as well).
The second line segment is drawn vertically from the x-axis, upward, has length 2, and thus ends with y-coordinate 2.
The fourth line segment is drawn vertically upward, has length 4 and thus ends with a y-coordinate of $2 + 4 = 6$.
The sixth line segment is drawn vertically upward, has length 6 and thus ends with a y-coordinate of $2 + 4 + 6 = 12$.
The endpoint of the 45th line segment has y-coordinate equal to $2 + 4 + 6 + \cdots + 40 + 42 + 44$.
Rearranging the terms of this sum, we get
$$(2 + 44) + (4 + 42) + (6 + 40) + \cdots + (20 + 26) + (22 + 24) = 46 \times 11 = 506.$$
Thus, $(529, 506)$ is the endpoint of the 45th line segment.
The 46th line segment is vertical and has length 46.
Therefore the y-coordinate of the endpoint of the 46th line segment is
$$2 + 4 + 6 + \cdots + 40 + 42 + 44 + 46 = 506 + 46 = 552.$$
Since the x-coordinate of the endpoint of the 46th line segment is the same as that of the 45th, then the endpoint of the next line segment that Paul draws is $(529, 552)$.

ANSWER: (A)

Grade 8

1. The number $10\,101$ is ten thousand one hundred one.
   Therefore, it is equal to $10\,000 + 100 + 1$.

   ANSWER: (D)

2. Since one scoop can feed 8 goldfish, then 4 scoops can feed $4 \times 8 = 32$ goldfish.

   ANSWER: (E)

3. Evaluating, $(2014 - 2013) \times (2013 - 2012) = 1 \times 1 = 1$.

   ANSWER: (B)

4. The measure of the three angles in any triangle add to $180°$.
   Since two of the angles measure $90°$ and $55°$, then the third angle in the triangle measures
   $180° - 90° - 55° = 35°$.
   The measure of the smallest angle in the triangle is $35°$.

   ANSWER: (D)

5. The positive value of any number is equal to the distance that the number is away from zero
   along a number line.
   The smaller the distance a number is away from zero along a number line, the closer the number
   is to zero.
   Ignoring negative signs (that is, we consider all 5 answers to be positive), the number that is
   closest to zero is the smallest number.
   The smallest number from the list $\{1101, 1011, 1010, 1001, 1110\}$ is 1001.
   Of the given answers, the number -1001 is the closest number to zero.

   ANSWER: (D)

6. Since $5y - 100 = 125$, then $5y = 125 + 100 = 225$, and so $y = \frac{225}{5} = 45$.

   ANSWER: (A)

7. Each prime number is divisible by exactly two numbers, 1 and itself.
   Each of the even numbers $\{12, 14, 16, 18, 20, 22, 24, 26, 28\}$ is divisible by 1 and itself and also
   by 2.
   Therefore the numbers $\{12, 14, 16, 18, 20, 22, 24, 26, 28\}$ are not prime.
   Each of the numbers $\{15, 21, 27\}$ is divisible by 1 and itself and also by 3.
   Therefore the numbers $\{15, 21, 27\}$ are not prime.
   The number 25 is divisible by 1 and itself and also by 5, so it is not prime.
   Each of the remaining numbers is divisible by exactly 1 and itself.
   Thus, the list of prime numbers between 10 and 30 is $\{11, 13, 17, 19, 23, 29\}$.
   There are 6 prime numbers between 10 and 30.

   ANSWER: (C)

8. The given triangle is isosceles, so the unmarked side also has length x cm.
   The perimeter of the triangle is 53 cm, so $x + x + 11 = 53$ or $2x + 11 = 53$ or $2x = 53 - 11 = 42$,
   and so $x = \frac{42}{2} = 21$.

   ANSWER: (B)

9. *Solution 1*

   To order the fractions $\left\{\frac{3}{7}, \frac{3}{2}, \frac{6}{7}, \frac{3}{5}\right\}$ from smallest to largest, we first express each fraction with a common denominator of $7 \times 2 \times 5 = 70$.

   The set $\left\{\frac{3}{7}, \frac{3}{2}, \frac{6}{7}, \frac{3}{5}\right\}$ is equivalent to the set $\left\{\frac{3\times10}{7\times10}, \frac{3\times35}{2\times35}, \frac{6\times10}{7\times10}, \frac{3\times14}{5\times14}\right\}$ or to the set $\left\{\frac{30}{70}, \frac{105}{70}, \frac{60}{70}, \frac{42}{70}\right\}$.

   Ordered from smallest to largest, the set is $\left\{\frac{30}{70}, \frac{42}{70}, \frac{60}{70}, \frac{105}{70}\right\}$, so ordered from smallest to largest the original set is $\left\{\frac{3}{7}, \frac{3}{5}, \frac{6}{7}, \frac{3}{2}\right\}$.

   Solution 2

   Since the numerators of the given fractions are each either 3 or 6, we rewrite each fraction with a common numerator of 6.

   The set $\left\{\frac{3}{7}, \frac{3}{2}, \frac{6}{7}, \frac{3}{5}\right\}$ is equivalent to the set $\left\{\frac{3\times2}{7\times2}, \frac{3\times2}{2\times2}, \frac{6}{7}, \frac{3\times2}{5\times2}\right\}$ or to the set $\left\{\frac{6}{14}, \frac{6}{4}, \frac{6}{7}, \frac{6}{10}\right\}$.

   Since the numerators are equal, then the larger the denominator, the smaller the fraction.

   Ordered from smallest to largest, the set is $\left\{\frac{6}{14}, \frac{6}{10}, \frac{6}{7}, \frac{6}{4}\right\}$, so ordered from smallest to largest the original set is $\left\{\frac{3}{7}, \frac{3}{5}, \frac{6}{7}, \frac{3}{2}\right\}$.

   ANSWER: (A)

10. *Solution 1*

    The ratio of the number of girls to the number of boys is $3 : 5$, so for every 3 girls there are 5 boys and $3 + 5 = 8$ students.

    That is, the number of girls in the class is $\frac{3}{8}$ of the number of students in the class.

    Since the number of students in the class is 24, the number of girls is $\frac{3}{8} \times 24 = \frac{72}{8} = 9$.

    The remaining $24 - 9 = 15$ students in the class must be boys.

    Therefore, there are $15 - 9 = 6$ fewer girls than boys in the class.

    Solution 2

    The ratio of the number of girls to the number of boys is $3 : 5$, so if there are $3x$ girls in the class then there are $5x$ boys.

    In total, there are 24 students in the class, so $3x + 5x = 24$ or $8x = 24$ and so $x = 3$.

    The number of girls in the class is $3x = 3(3) = 9$ and the number of boys in the class is $5x = 5(3) = 15$.

    Therefore, there are $15 - 9 = 6$ fewer girls than boys in the class.

    Solution 3

    The ratio of the number of girls to the number of boys is $3 : 5$, so for every 3 girls there are 5 boys and $3 + 5 = 8$ students.

    That is, $\frac{3}{8}$ of the number of students in the class are girls and $\frac{5}{8}$ of the number of students are boys.

    Thus, the difference between the number of boys in the class and the number of girls in the class is $\frac{5}{8} - \frac{3}{8} = \frac{2}{8} = \frac{1}{4}$ of the number of students in the class.

    Therefore, there are $\frac{1}{4} \times 24 = 6$ fewer girls than boys in the class.

    ANSWER: (D)

11. Since there are 7 days in a week, any multiple of 7 days after a Wednesday is also a Wednesday.

    Since 70 is a multiple of 7, then 70 days after John was born, the day is also a Wednesday.

    Therefore, 71 days after John was born is a Thursday and 72 days after John was born is a Friday.

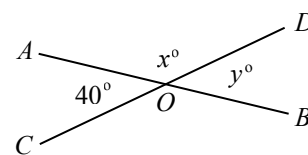   Alison was born on a Friday.

    ANSWER: (E)

12. Opposite angles have equal measures.
    Since $\angle AOC$ and $\angle DOB$ are opposite angles, then $y = 40$.
    Straight angle COD measures $180°$.
    Since $\angle AOC + \angle AOD = \angle COD$, then $40° + x° = 180°$ and so
    $x = 140$.
    The difference $x - y$ is $140 - 40 = 100$.

    Answer: (E)

13. *Solution 1*
    Since the scores in each of the 5 sets are listed in order, from smallest to largest, the number
    in the middle (the 3rd number) of the 5 numbers is the median.
    The mean is calculated and also shown below for each of the 5 sets of scores.

Set of Scores	Median	Mean
$10, 20, 40, 40, 40$	40	$\dfrac{10 + 20 + 40 + 40 + 40}{5} = \dfrac{150}{5} = 30$
$40, 50, 60, 70, 80$	60	$\dfrac{40 + 50 + 60 + 70 + 80}{5} = \dfrac{300}{5} = 60$
$20, 20, 20, 50, 80$	20	$\dfrac{20 + 20 + 20 + 50 + 80}{5} = \dfrac{190}{5} = 38$
$10, 20, 30, 100, 200$	30	$\dfrac{10 + 20 + 30 + 100 + 200}{5} = \dfrac{360}{5} = 72$
$50, 50, 50, 50, 100$	50	$\dfrac{50 + 50 + 50 + 50 + 100}{5} = \dfrac{300}{5} = 60$

    From the table above, we see that the first set of scores is the only set in which the median,
    40, is greater than the mean, 30.

    Solution 2
    The first set of 5 scores, $10, 20, 40, 40, 40$, is ordered from smallest to largest and thus the median
    is the 3rd (middle) number of this set, 40.
    The mean of the final 3 numbers of this set is also 40, since each is equal to 40.
    Since the first 2 numbers of the set are both less than the mean of the other 3 numbers (40),
    then the mean of all 5 numbers in the set must be less than 40.
    Using a similar argument for the remaining four answers, we see that each of the other four
    sets has its mean equal to or greater than its median.
    Therefore, the first set of scores is the only set in which the median is greater than the mean.
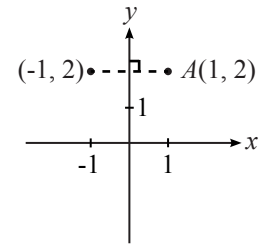
    Answer: (A)

14. Betty has 3 equally likely choices for the flavour of ice cream (chocolate or vanilla or strawberry).
    For each of these 3 choices, she has 2 equally likely choices for the syrup (butterscotch or fudge).
    Thus, Betty has 3×2 or 6 equally likely choices for the flavour and syrup of the sundae.
    For each of these 6 choices, Betty has 3 equally likely choices for the topping (cherry or banana
    or pineapple).
    This makes 6×3 or 18 equally likely choices for her sundae.
    Since only 1 of these 18 choices is the sundae with vanilla ice cream, fudge syrup and banana
    topping, then the probability that Betty randomly chooses this sundae is $\frac{1}{18}$.

    Answer: (A)

15. After reflecting the point $A(1, 2)$ in the y-axis, the y-coordinate of
the image will be the same as the y-coordinate of point A, $y = 2$.
Point A is a distance of 1 to the right of the y-axis.
The image will be the same distance from the y-axis, but to the
left of the y-axis. Thus, the image has x-coordinate -1.
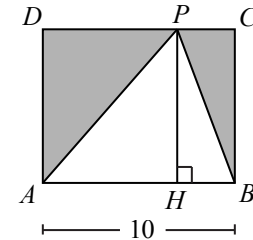The coordinates of the reflected point are $(-1, 2)$.

ANSWER: (B)

16. *Solution 1*
Construct PH perpendicular to AB, as shown.
The area of $\triangle ABP$ is 40 and so $\frac{1}{2} \times AB \times PH = 40$, or
$AB \times PH = 80$, and since $AB = 10$, then $PH = 8$.
Since $CB = PH = 10$, the area of $ABCD$ is $10 \times 8 = 80$.
The shaded area equals the area of $\triangle ABP$ subtracted from
the area of $ABCD$, or $80 - 40 = 40$.

Solution 2
As in Solution 1, we construct PH perpendicular to AB.
Since both DA and CB are perpendicular to AB, then PH is parallel to DA and to CB.
That is, $DAHP$ and $PHBC$ are rectangles.
Diagonal PA divides the area of rectangle $DAHP$ into two equal areas, $\triangle PAH$ and $\triangle PAD$.
Diagonal PB divides the area of rectangle $PHBC$ into two equal areas, $\triangle PBH$ and $\triangle PBC$.
Therefore, the area of $\triangle PAH$ added to the area of $\triangle PBH$ is equal to the area of $\triangle PAD$
added to the area of $\triangle PBC$.
However, the area of $\triangle PAH$ added to the area of $\triangle PBH$ is equal to the area of $\triangle ABP$, which
is 40.
So the area of $\triangle PAD$ added to the area of $\triangle PBC$ is also 40.
Therefore, the area of the shaded region is 40.

ANSWER: (B)

17. Of the 10 multiple choice questions, Janine got 80% or $0.80 \times 10 = 8$ correct.
Of the 30 short answer questions, Janine got 70% or $0.70 \times 30 = 21$ correct.
In total, Janine answered $8 + 21 = 29$ of the 40 questions correct, or $\frac{29}{40} \times 100\% = 72.5\%$ correct.

ANSWER: (B)

18. The area of the rectangle, 48 cm^2, is given by the product of the rectangle's length and width,
and so we must consider all possible pairs of whole numbers whose product is 48.
In the table below, we systematically examine all possible factors of 48 in order to determine
the possible whole number side lengths of the rectangle and its perimeter.

Factors of 48	Side Lengths of Rectangle	Perimeter of Rectangle
$48 = 1 \times 48$	1 and 48	$2 \times (1 + 48) = 2 \times 49 = 98$
$48 = 2 \times 24$	2 and 24	$2 \times (2 + 24) = 2 \times 26 = 52$
$48 = 3 \times 16$	3 and 16	$2 \times (3 + 16) = 2 \times 19 = 38$
$48 = 4 \times 12$	4 and 12	$2 \times (4 + 12) = 2 \times 16 = 32$
$48 = 6 \times 8$	6 and 8	$2 \times (6 + 8) = 2 \times 14 = 28$

The side lengths of the rectangle having area 48 cm^2 and perimeter 32 cm, are 4 and 12.
In cm, the positive difference between the length and width of the rectangle is $12 - 4 = 8$.

ANSWER: (D)

19. At Store Q, the bicycle's regular price is 15% more than the price at Store P, or 15% more than $200.

    Since 15% of 200 is $\frac{15}{100} \times 200 = 0.15 \times 200 = 30$, then 15% more than $200 is $200+$30 or $230.

    This bicycle is on sale at Store Q for 10% off of the regular price, $230.

    Since 10% of 230 is $\frac{10}{100} \times 230 = 0.10 \times 230 = 23$, then 10% off of $230 is $230 − $23 or $207.

    The sale price of the bicycle at Store Q is $207.   ANSWER: (D)

20. Using 7 of the 5¢ stamps (35¢) and 1 of the 8¢ stamps (8¢) makes 43¢ in postage.

    Also, 3 of the 5¢ stamps (15¢) and 3 of the 8¢ stamps (24¢) makes 39¢ in postage.

    This eliminates the choices (D) and (E).

    Of the five given answers, the next largest is 27¢.

    To make 27¢ in postage, $0, 1, 2,$ or 3 of the 8¢ stamps must be used (using more than 3 of the 8¢ stamps would exceed 27¢ in postage).

    If 0 of the 8¢ stamps are used, then the remaining 27¢ must be made from 5¢ stamps.

    This is not possible since 27 is not a multiple of 5.

    If 1 of the 8¢ stamps are used, then the remaining 27¢ − 8¢ = 19¢ must be made from 5¢ stamps. This is not possible since 19 is not a multiple of 5.

    If 2 of the 8¢ stamps are used, then the remaining 27¢ − 16¢ = 11¢ must be made from 5¢ stamps. This is not possible since 11 is not a multiple of 5.

    If 3 of the 8¢ stamps are used, then the remaining 27¢ − 24¢ = 3¢ must be made from 5¢ stamps. This is clearly not possible.

    Therefore of the five given answers, 27¢ is the largest amount of postage that cannot be made using only 5¢ and 8¢ stamps.

    It is worth noting that all amounts of postage larger than 27¢ can be made using only 5¢ and 8¢ stamps.

    To see why this is true, first consider that all five postage amounts from 28¢ to 32¢ can be made as shown in the table below.

Number of 5¢ Stamps	Number of 8¢ Stamps	Value of Stamps
4	1	$(4 \times 5¢)+(1 \times 8¢)= 28¢$
1	3	$(1 \times 5¢)+(3 \times 8¢)= 29¢$
6	0	$(6 \times 5¢)+(0 \times 8¢)= 30¢$
3	2	$(3 \times 5¢)+(2 \times 8¢)= 31¢$
0	4	$(0 \times 5¢)+(4 \times 8¢)= 32¢$

    The next five amounts of postage, 33¢ to 37¢, can be made by adding 1 additional 5¢ stamp to each of the previous five amounts of postage.

    That is, 28¢ + 5¢ = 33¢, and 29¢ + 5¢ = 34¢, and so on.

    We can make every postage amount larger than 27¢ by continuing in this way to add 1 additional 5¢ stamp to each of the five amounts from the previous group.

    ANSWER: (C)

21. The shaded area in the top three rows of the diagram contains 6 circles with radius 1 cm.

    The shaded area in the bottom row of the diagram contains 4 semi-circles with radius 1 cm, whose combined area is equal to the area of 2 circles with radius 1 cm.

    In total, the shaded area of the diagram contains $6 + 2 = 8$ circles with radius 1 cm.

    In cm², the total shaded area is $8 \times \pi \times 1^2 = 8\pi$.

    ANSWER: (E)

22. We first determine the surface area of the original cube, before the two cubes were cut from its corners.

    The original cube had 6 identical square faces, each of whose area was $3 \times 3 = 9$ cm^2.

    Thus, the surface area of the original cube was $6 \times 9 = 54$ cm^2.

    Next we will explain why the resulting solid in question has surface area equal to that of the original cube, 54 cm^2.

    Consider the front, bottom right corner of the resulting solid, as shown and labeled in Figure 1.

    Cutting out a 1 cm by 1 cm by 1 cm cube from this corner exposes 3 new square faces, $RSPQ$, $RSTU$ and $SPWT$, which were not a part of the surface area of the original cube.

    Next we consider what surface area was part of the original cube that is not present in the resulting solid.

    Figure 1

    To visualize this, we replace the vertex V of the original cube and reconstruct segments QV, UV and WV, as shown in Figure 2.

    We note that $PQRSTUVW$ is the 1 cm by 1 cm by 1 cm cube that was cut from the corner of the original cube.

    In Figure 2, we see that the surface areas that were part of the original cube that are not present in the resulting solid are the 3 square faces $UTWV$, $QPWV$ and $RQVU$.
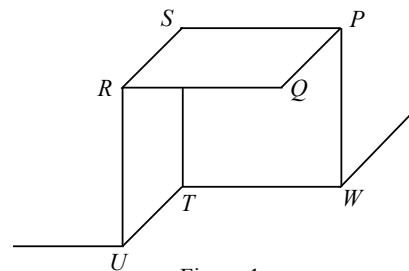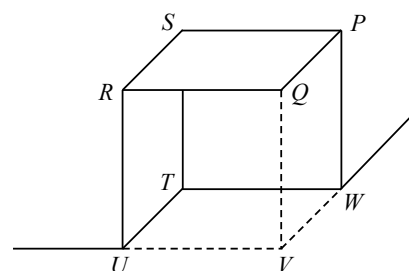

    Figure 2

    To summarize, removal of the 1 cm by 1 cm by 1 cm cube from the corner of the original cube exposes 3 new square faces, $RSPQ$, $RSTU$ and $SPWT$, which were not a part of the surface area of the original cube.

    These 3 faces represent half of the surface area of cube $PQRSTUVW$ (since they are 3 of the 6 identical faces of the cube).

    However, in removing the 1 cm by 1 cm by 1 cm cube from the corner of the original cube, the 3 faces, $UTWV$, $QPWV$ and $RQVU$, are lost.

    These 3 faces represent the other half of the surface area of cube $PQRSTUVW$.

    That is, the total surface area gained by removal of the 1 cm by 1 cm by 1 cm cube is equal to the total surface area lost by removing this same cube.

    This same argument can be made for the removal of the 2 cm by 2 cm by 2 cm cube from the top, back left corner of the original cube.

    Thus the surface area of the resulting solid is equal to the surface area of the original cube, which was 54 cm^2.

    ANSWER: (E)

23. The first positive odd integer is 1, the second is $2(2) - 1 = 3$, the third is $2(3) - 1 = 5$, the fourth is $2(4) - 1 = 7$.

    That is, the one hundredth positive odd integer is $2(100) - 1 = 199$ and the sum that we are being asked to determine is $1 + 3 + 5 + \cdots + 195 + 197 + 199$.

    Solution 1

    Since each odd integer can be expressed as the sum of two consecutive integers, we rewrite $1 + 3 + 5 + \cdots + 195 + 197 + 199$ as $1 + (1 + 2) + (2 + 3) + \cdots + (97 + 98) + (98 + 99) + (99 + 100)$.

    Rearranging the terms in the previous sum, we get

    $(1 + 2 + 3 + \cdots + 98 + 99 + 100) + (1 + 2 + 3 + \cdots + 97 + 98 + 99)$.

The sum in the first set of brackets, $1 + 2 + 3 + \cdots + 98 + 99 + 100$ is equal to 5050.
The sum in the second set of brackets, $1 + 2 + 3 + \cdots + 98 + 99$ is 100 less than 5050 or 4950.
Therefore, $1 + 3 + 5 + \cdots + 195 + 197 + 199 = 5050 + 4950 = 10\,000$.

Solution 2
Doubling each term on the left of the equality $1 + 2 + 3 + \cdots + 98 + 99 + 100 = 5050$, doubles the result on the right.
That is, $2 + 4 + 6 + \cdots + 196 + 198 + 200 = 10\,100$.
Subtracting 1 from each term on the left side of this equality gives
$(2-1) + (4-1) + (6-1) + \cdots + (196-1) + (198-1) + (200-1)$ or $1 + 3 + 5 + \cdots + 195 + 197 + 199$, the required sum.
Since there are 100 terms on the left side of the equality, then subtracting one 100 times reduces the left side of the equality by 100.
Subtracting 100 from the right side, we get $10\,100 - 100 = 10\,000$, and so
$1 + 3 + 5 + \cdots + 195 + 197 + 199 = 10\,000$.

Solution 3
It is also possible to determine the sum of the first 100 positive odd integers,
$1 + 3 + 5 + \cdots + 195 + 197 + 199$, without using the result given in the question.
Reorganizing the terms into sums of pairs, first the largest number with the smallest, then the next largest number with the next smallest, and so on,
$1 + 3 + 5 + \cdots + 195 + 197 + 199 = (1+199) + (3+197) + (5+195) + \cdots + (97+103) + (99+101)$.
The sum of the first pair, $1 + 199$ is 200.
Each successive pair following $1 + 199$ includes a number that is 2 more than a number in the previous pair, and also a number that is 2 less than a number in the previous pair, so then the sum of every pair is also 200.
The total number of terms in the sum is 100, and thus there are 50 pairs of numbers each of which has a sum of 200.
Therefore, $1 + 3 + 5 + \cdots + 195 + 197 + 199 = 50 \times 200 = 10\,000$.
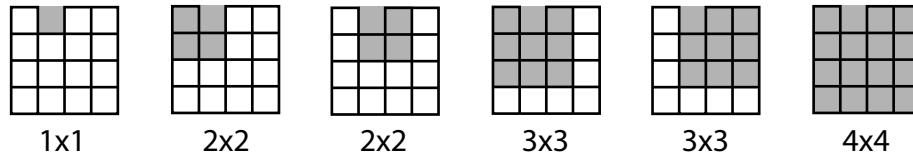
ANSWER: (B)

24. The original 4×4 grid contains exactly 30 squares, of which 16 are of size 1×1, 9 are of size 2×2, 4 are of size 3×3, and 1 is of size 4×4 ($16 + 9 + 4 + 1 = 30$).
In each of the 5 answers given, exactly one or two of the 1×1 squares are missing from the original grid.
We determine the number of these 30 squares that cannot be formed as a result of these missing 1×1 squares, and subtract this total from 30 to find the total number of squares in each of the 5 given configurations.

	Number of Missing Squares				Total Number of Squares
	1×1	2×2	3×3	4×4	
A	1	1	1	1	$30 - 4 = 26$
B	1	2	2	1	$30 - 6 = 24$
C	2	2	2	1	$30 - 7 = 23$
D	2	2	2	1	$30 - 7 = 23$
E	2	2	2	1	$30 - 7 = 23$

For clarity, the 6 squares that exist in the original grid but that are missing from the grid in answer B are shown below by size.

| 1x1 | 2x2 | 2x2 | 3x3 | 3x3 | 4x4 |

The grid given in answer B contains exactly 24 squares.

ANSWER: (B)

25. A total of N people participated in the survey.

Exactly $\frac{9}{14}$ of those surveyed, or $\frac{9}{14} \times N$ people, said that the colour of the flower was important.

Exactly $\frac{7}{12}$ of those surveyed, or $\frac{7}{12} \times N$ people, said that the smell of the flower was important.

Since the number of people surveyed must be a whole number, then N must be a multiple of 14 and N must also be a multiple of 12.

The lowest common multiple of 14 and 12 is $2 \times 7 \times 6$ or 84, and so N is a multiple of 84.

Since N is a multiple of 84, let $N = 84k$ where k is some positive integer.

That is, $\frac{9}{14} \times N = \frac{9}{14}(84k) = 54k$ people said that the colour of the flower was important and $\frac{7}{12} \times N = \frac{7}{12}(84k) = 49k$ people said that the smell of the flower was important.

The information can be represented with a Venn diagram, as shown below.
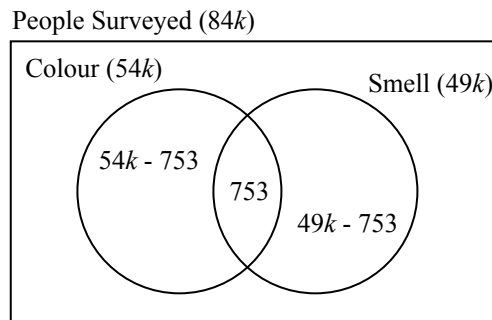
Since $N = 84k$, we must find both a maximum and a minimum value for k in order to determine the number of possible values for N.

Finding a Minimum Value for k

We know that 753 people said that both the colour and the smell were important.

We also know that $54k$ said that the colour was important and that $49k$ said that the smell was important.

The 753 people who said that both were important are included among both the $54k$ people and among the $49k$ people.

Therefore, $54k$ must be at least 753 and $49k$ must also be at least 753.

Since $54k$ is larger than $49k$, then we only need to find k for which $49k$ is larger than or equal to 753.

We note that $49 \times 15 = 735$ and that $49 \times 16 = 784$.

Therefore, the smallest value of k for which $49k$ is at least 753 is $k = 16$.

As noted above, $k = 16$ will also make $54k$ larger than 753.

Therefore, the minimum possible value for k is 16.

Finding a Maximum Value for k

There were $84k$ people surveyed of whom $54k$ said that the colour was important.
This means that there are $84k - 54k = 30k$ people who said that the colour was not important.
We note that $49k$ people said that the smell was important. These $49k$ people include some or all of the $30k$ people who said that the colour was not important.
In other words, at most $30k$ of the $49k$ people said that the smell was important and the colour was not important, which means that at least $49k - 30k = 19k$ people said that the smell was important and that the colour was important.
Since we know that 753 people said that both the smell and colour were important, then 753 is at least as large as $19k$.
Since $19 \times 40 = 760$ and $19 \times 39 = 741$, then k must be at most 39.

The minimum value of k is 16 and the maximum value of k is 39.
Thus, there are $39 - 16 + 1 = 24$ possible values for k and since $N = 84k$, there are also 24 possible values for N.

Note: We should also justify that each value of k between 16 and 39 is possible.
Using the Venn diagram above, we see that the total of $84k$ people surveyed means that $84k - (54k - 753) - 753 - (49k - 753) = 753 - 19k$ people fall outside the two circles.
From our work in finding the minimum above, we can see that for each k between 16 and 39, inclusive, each of the three integers $54k - 753$, 753, and $49k - 753$ is positive, since k is at least 16. Also, from our work in finding the maximum above, we can see that for each k between 16 and 39, inclusive, the integer $753 - 19k$ is positive, since k is at most 39.
Therefore, all four quantities are positive integers for each of these values of k, which means that we can construct a Venn diagram with these numbers, as required.

ANSWER: (D)