The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING
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Galois Contest

(Grade 10)

Wednesday, April 16, 2014
(in North America and South America)

Thursday, April 17, 2014
(outside of North America and South America)

Do not open this booklet until instructed to do so.

Time: 75 minutes | **Number of questions:** 4
Calculators are permitted | **Each question is worth 10 marks**

Parts of each question can be of two types:

1. **SHORT ANSWER** parts indicated by
   - worth 2 or 3 marks each
   - full marks given for a correct answer which is placed in the box
   - **part marks awarded only if relevant work** is shown in the space provided
2. **FULL SOLUTION** parts indicated by
   - worth the remainder of the 10 marks for the question
   - **must be written in the appropriate location** in the answer booklet
   - marks awarded for completeness, clarity, and style of presentation
   - a correct solution poorly presented will not earn full marks

WRITE ALL ANSWERS IN THE ANSWER BOOKLET PROVIDED.

- Extra paper for your finished solutions supplied by your supervising teacher must be inserted into your answer booklet. Write your name, school name, and question number on any inserted pages.
- Express calculations and answers as exact numbers such as $\pi + 1$ and $\sqrt{2}$, etc., rather than as $4.14\ldots$ or $1.41\ldots$, except where otherwise indicated.

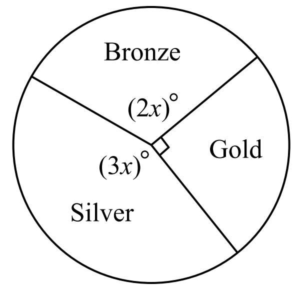
Do not discuss the problems or solutions from this contest online for the next 48 hours.

The name, grade, school and location of some top-scoring students will be published on our Web site, http://www.cemc.uwaterloo.ca. In addition, the name, grade, school and location, and score of some top-scoring students may be shared with other mathematical organizations for other recognition opportunities.

TIPS:

1. Please read the instructions on the front cover of this booklet.
2. Write all answers in the answer booklet provided.
3. For questions marked , place your answer in the appropriate box in the answer booklet and **show your work**.
4. For questions marked , provide a well-organized solution in the answer booklet. Use mathematical statements and words to explain all of the steps of your solution. Work out some details in rough on a separate piece of paper before writing your finished solution.
5. Diagrams are *not* drawn to scale. They are intended as aids only.

1. The pie chart shows the distribution of the number of bronze, silver and gold medals in a school's trophy case.



   (a) What is the value of x?

   (b) Write the ratio of the number of bronze medals to the number of silver medals to the number of gold medals in lowest terms.

   (c) If there is a total of 80 medals in the trophy case, determine the number of bronze medals, the number of silver medals, and the number of gold medals in the trophy case.

   (d) The trophy case begins with the same number of each type of medal as in part (c). A teacher then finds a box with medals and adds them to the trophy case. The ratio of the number of bronze medals, to the number of silver medals, to the number of gold medals is unchanged. What is the smallest number of medals that could now be in the trophy case?

2. An airplane holds a maximum of 245 passengers. To accommodate the extra expense of transporting luggage, passengers are charged a baggage fee of \$20 for the first bag checked plus \$7 for each additional bag checked. (Passengers who do not check a bag are not charged a baggage fee.)

   (a) On one flight, 200 passengers checked exactly one bag and the other 45 passengers checked exactly two bags. Determine the total of the baggage fees for checked bags.

   (b) On a second flight, the plane was again completely full. Every passenger checked exactly one or two bags. If a total of \$5173 in baggage fees were collected, how many passengers checked exactly two bags?

   (c) On a third flight, exactly \$6825 was collected in baggage fees. Explain why there must be at least one passenger who checked at least three bags.

   (d) On a fourth flight, exactly \$142 was collected in baggage fees. Explain why there must be at least one passenger who checked at least three bags.

3. Cards in a deck are numbered consecutively with positive integers. The cards are selected in pairs, (a, b) with $a < b$, to create a given sum, $a + b$. For example, Anna has a set of cards numbered from 1 to 50 and she is required to create a sum of 60. Two of the pairs that she can select are $(10, 50)$ and $(25, 35)$.

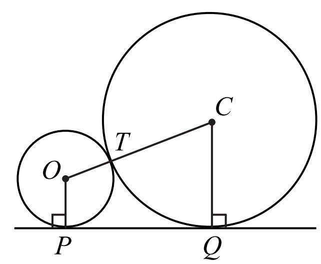
(a) Emily has a set of 10 cards numbered consecutively from 1 to 10. There are exactly 3 pairs that she can select, each having a sum of 8. List the 3 pairs.

(b) Silas has a set of 10 cards numbered consecutively from 1 to 10. Determine the number of pairs that he can select with a sum of 13.

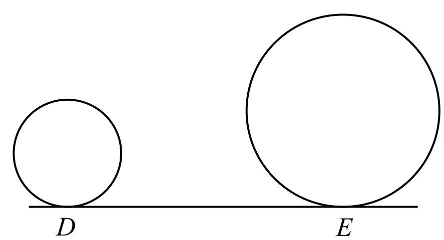
(c) Daniel has a set of k cards numbered consecutively from 1 to k. He can select exactly 10 pairs that have a sum of 100. What is the value of k?

(d) Derrick has a set of 75 cards numbered consecutively from 1 to 75. He can select exactly 33 pairs that have a sum of S. Determine, with justification, all possible values of S.

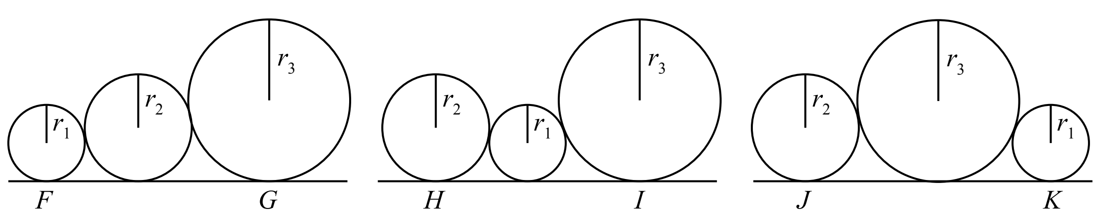
4. (a) A circle of radius 2 and a circle of radius 5 are externally tangent to each other at T and tangent to a horizontal line at points P and Q, as shown. If points O and C are the centres of the circles, then O, T, C are collinear and both OP and CQ are perpendicular to PQ. By constructing a line segment passing through O and parallel to PQ, determine the distance between P and Q.



(b) A circle of radius 4 and a circle of radius 9 are tangent to a horizontal line at points D and E, as shown. A third circle can be placed between these two circles so that it is externally tangent to each circle and tangent to the horizontal line. If $DE = 24$, determine with justification, the radius of this third circle.



(c) Three circles with radii $r_1 < r_2 < r_3$ are placed so that they are tangent to a horizontal line, and so that adjacent circles are externally tangent to each other. F, G, H, I, J, and K are the points of tangency of the circles to the horizontal line, as shown. The lengths of FG, HI, JK, in no particular order, are 18, 20 and 22. Determine, with justification, the values of r_1, r_2 and r_3.



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In 2013, more than 15 000 students from around the world registered to write the Fryer, Galois and Hypatia Contests.

Encourage your teacher to register you for the Canadian Intermediate Mathematics Contest or the Canadian Senior Mathematics Contest, which will be written in November 2014.

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