The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING

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Fryer Contest

(Grade 9)

Wednesday, April 16, 2014
(in North America and South America)

Thursday, April 17, 2014
(outside of North America and South America)

Do not open this booklet until instructed to do so.

Time: 75 minutes
Calculators are permitted

Number of questions: 4
Each question is worth 10 marks

Parts of each question can be of two types:

1. **SHORT ANSWER** parts indicated by
   - worth 2 or 3 marks each
   - full marks given for a correct answer which is placed in the box
   - **part marks awarded only if relevant work** is shown in the space provided
2. **FULL SOLUTION** parts indicated by
   - worth the remainder of the 10 marks for the question
   - **must be written in the appropriate location** in the answer booklet
   - marks awarded for completeness, clarity, and style of presentation
   - a correct solution poorly presented will not earn full marks

WRITE ALL ANSWERS IN THE ANSWER BOOKLET PROVIDED.

- Extra paper for your finished solutions supplied by your supervising teacher must be inserted into your answer booklet. Write your name, school name, and question number on any inserted pages.
- Express calculations and answers as exact numbers such as $\pi + 1$ and $\sqrt{2}$, etc., rather than as $4.14\ldots$ or $1.41\ldots$, except where otherwise indicated.

Do not discuss the problems or solutions from this contest online for the next 48 hours.

The name, grade, school and location of some top-scoring students will be published on our Web site, http://www.cemc.uwaterloo.ca. In addition, the name, grade, school and location, and score of some top-scoring students may be shared with other mathematical organizations for other recognition opportunities.

TIPS:

1. Please read the instructions on the front cover of this booklet.
2. Write all answers in the answer booklet provided.
3. For questions marked, place your answer in the appropriate box in the answer booklet and **show your work**.
4. For questions marked, provide a well-organized solution in the answer booklet. Use mathematical statements and words to explain all of the steps of your solution. Work out some details in rough on a separate piece of paper before writing your finished solution.
5. Diagrams are *not* drawn to scale. They are intended as aids only.

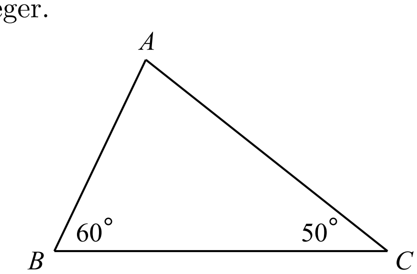
1. (a) The positive integers from 1 to 99 are written in order next to each other to form the integer 123456789101112 . . . 9899. How many digits does this integer have?

   (b) The positive integers from 1 to 199 are written in order next to each other to form the integer 123456789101112 . . . 198199. How many digits does this integer have?

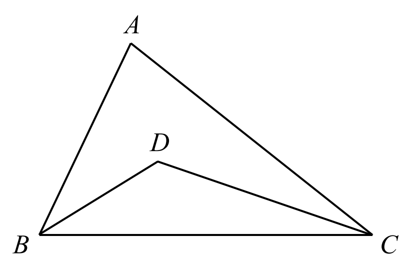
   (c) The positive integers from 1 to n are written in order next to each other. If the resulting integer has 1155 digits, determine n.

   (d) The positive integers from 1 to 1000 are written in order next to each other. Determine the 1358^{th} digit of the resulting integer.

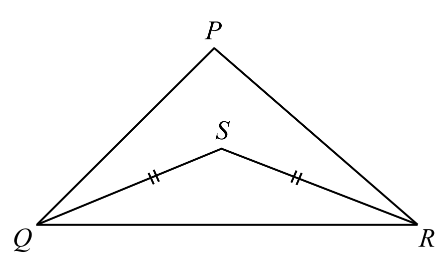
2. (a) In $\triangle ABC$, $\angle ABC = 60^\circ$ and $\angle ACB = 50^\circ$. What is the measure of $\angle BAC$?



   (b) An *angle bisector* is a line segment that divides an angle into two equal angles. In $\triangle ABC$, $\angle ABC = 60^\circ$ and $\angle ACB = 50^\circ$. If BD and CD are angle bisectors of $\angle ABC$ and $\angle ACB$, respectively, what is the measure of $\angle BDC$?

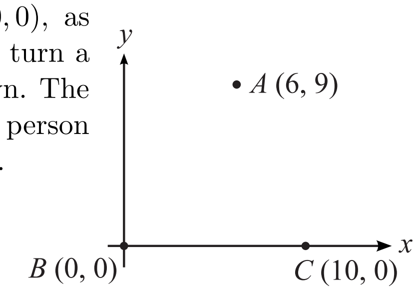


   (c) Point S is inside $\triangle PQR$ so that QS and RS are angle bisectors of $\angle PQR$ and $\angle PRQ$, respectively, with $QS = RS$. If $\angle QSR = 140^\circ$, determine with justification, the measure of $\angle QPR$.



   (d) In $\triangle PQR$, QS and RS are angle bisectors of $\angle PQR$ and $\angle PRQ$, respectively, with $QS = RS$ (as in part (c)). Explain why it is not possible that $\angle QSR = 80^\circ$.

3. Triangle ABC begins with vertices $A(6,9), B(0,0), C(10,0)$, as shown. Two players play a game using $\triangle ABC$. On each turn a player can move vertex A one unit, either to the left or down. The x- and y-coordinates of A cannot be made negative. The person who makes the area of $\triangle ABC$ equal to 25 wins the game.



(a) What is the area of $\triangle ABC$ before the first move in the game is made?

(b) Dexter and Ella play the game. After several moves have been made, vertex A is at $(2,7)$. It is now Dexter's turn to move. Explain how Ella can always win the game from this point.

(c) Faisal and Geoff play the game, with Faisal always going first. There is a *winning strategy* for one of these players; that is, by following the rules in a certain way, he can win the game every time no matter how the other player plays.

(i) Which one of the two players has a winning strategy?

(ii) Describe a winning strategy for this player.

(iii) Justify why this winning strategy described in (ii) always results in a win.

4. The set $A = \{1,2\}$ has exactly four subsets: $\{\}$, $\{1\}$, $\{2\}$, and $\{1,2\}$. The four subset sums of A are 0, 1, 2 and 3 respectively. The sum of the subset sums of A is $0+1+2+3=6$. Note that $\{\}$ is the empty set and $\{1,2\}$ is the same as $\{2,1\}$.

(a) The set $\{1,2,3\}$ has exactly eight subsets and therefore it has eight subset sums. List all eight subset sums of $\{1,2,3\}$.

(b) Determine, with justification, the sum of all of the subset sums of $\{1,2,3,4,5\}$.

(c) Determine, with justification, the sum of all of the subset sums of $\{1,3,4,5,7,8,12,16\}$ that are divisible by 4.

2014 Fryer Contest (English)

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Encourage your teacher to register you for the Canadian Intermediate Mathematics Contest or the Canadian Senior Mathematics Contest, which will be written in November 2014.

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