

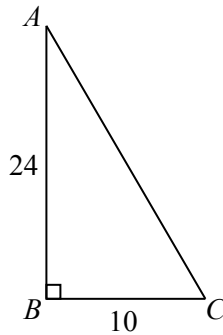


The CENTRE for EDUCATION  
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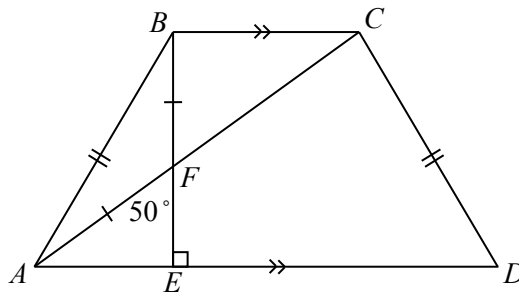
2014 Canadian Team Mathematics Contest

Team Problems

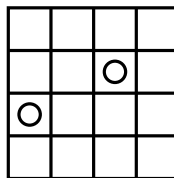
1. Beginning at 100, David counts down by 11 at a time. What is the first number less than 0 that David will count?
2. What is the value of  $1 + 2 + 3 + \cdots + 18 + 19 + 20$ ? (That is, what is the sum of the first 20 positive integers?)
3. In the diagram,  $\triangle ABC$  is right-angled at  $B$  with  $AB = 24$  and  $BC = 10$ . If  $AB$  and  $BC$  are each increased by 6, by how much does  $AC$  increase?



4. In the diagram,  $ABCD$  is a trapezoid with  $BC$  parallel to  $AD$  and  $AB = CD$ . Point  $E$  is on  $AD$  so that  $BE$  is perpendicular to  $AD$  and point  $F$  is the point of intersection of  $AC$  with  $BE$ . If  $AF = FB$  and  $\angle AFE = 50^\circ$ , what is the measure of  $\angle ADC$ ?



5. In the diagram below, how many rectangles contain both circles?

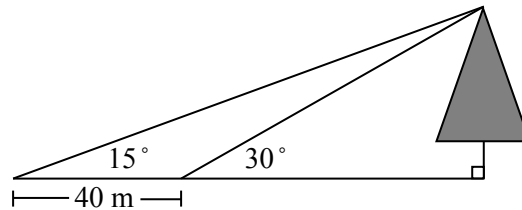


6. What is the smallest positive integer that is a multiple of each of 2, 4, 6, 8, 10, 12, 14, 16, 18, and 20?
7. Integers  $x$  and  $y$ , with  $x > y$ , satisfy  $x + y = 7$  and  $xy = 12$ .  
Integers  $m$  and  $n$ , with  $m > n$ , satisfy  $m + n = 13$  and  $m^2 + n^2 = 97$ .  
If  $A = x - y$  and  $B = m - n$ , determine the value of  $A - B$ .
8. Robert was born in the year  $n^2$ . On his birthday in the year  $(n + 1)^2$ , he will be 89 years old.  
In what year was he born?
9. Suppose that

$$\left(1 + \frac{1}{2}\right) \left(1 + \frac{1}{3}\right) \left(1 + \frac{1}{4}\right) \cdots \left(1 + \frac{1}{k}\right) \left(1 + \frac{1}{k+1}\right) = 2014$$

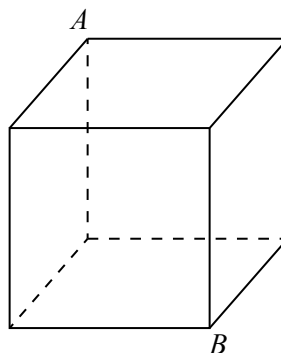
for some positive integer  $k$ . (There are  $k$  factors in the product.) What is the value of  $k$ ?

10. Rowena has a very long, level backyard. From a particular point, she determines that the angle of elevation to a large tree in her backyard is  $15^\circ$ . She moves 40 m closer and determines that the new angle of elevation is  $30^\circ$ . How tall, in metres, is the tree?



11. Line segment  $PQ$  has endpoints at  $P(6, -2)$  and  $Q(-3, 10)$ . Point  $R(a, b)$  is a point on  $PQ$  such that the distance from  $P$  to  $R$  is one-third the distance from  $P$  to  $Q$ . What is  $b - a$ ?
12. In how many ways can 105 be expressed as the sum of at least two consecutive positive integers?
13. An insect travels from vertex  $A$  to vertex  $B$  on the cube shown with edges of length 1 metre. The insect can travel in exactly one and only one of the following ways:
  - crawl along the edges of the cube (it crawls at 5 metres per minute along edges)
  - crawl along the faces of the cube (it crawls at 4 metres per minute along faces)
  - fly through the interior of the cube (it flies at 3 metres per minute along any path through the interior of the cube)

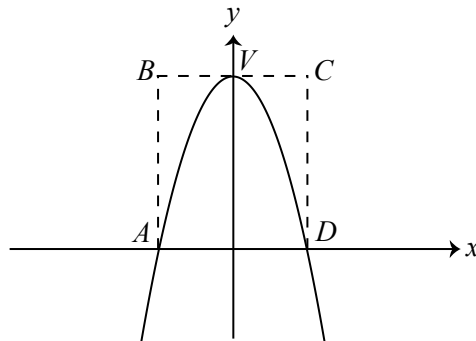
What is the shortest possible time, in minutes, that the insect can take to get from  $A$  to  $B$ ?



14. In an arithmetic sequence with 20 terms, the sum of the first three terms is 15 and the sum of the last three terms is 12. What is the sum of the 20 terms in the sequence?

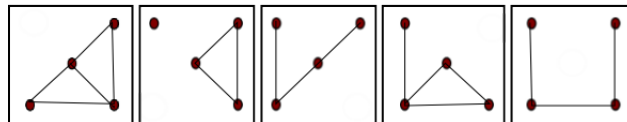
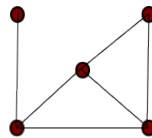
(An *arithmetic sequence* is a sequence in which each term after the first is obtained from the previous term by adding a constant. For example, 3, 5, 7, 9 are the first four terms of an arithmetic sequence.)

15. A parabola has equation  $y = k^2 - x^2$ , for some positive number  $k$ . Rectangle  $ABCD$  is drawn with sides parallel to the axes so that  $A$  and  $D$  are the points where the parabola intersects the  $x$ -axis and so that the vertex,  $V$ , of the parabola is the midpoint of  $BC$ . If the perimeter of the rectangle is 48, what is the value of  $k$ ?

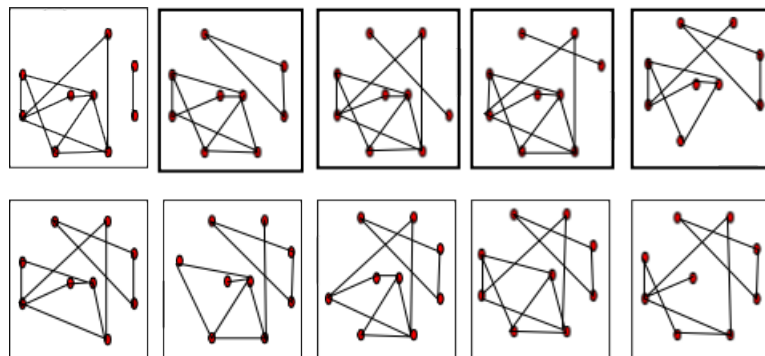


16. A *graph* is a diagram consisting of points (called *vertices*) and line segments joining some pairs of points (called *edges*).

Starting with an original graph with five vertices, as shown below, five new graphs are formed by removing one of the vertices from the original graph and all of the edges joined to this vertex.

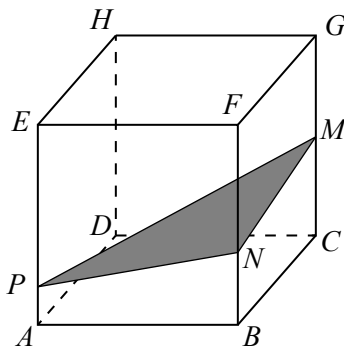


This process is repeated starting with an original graph with 10 vertices.



How many edges did this second original graph have?

17. A quantity of grey paint has a mass of 12 kg. The grey paint is a mixture of black paint and white paint, of which 80% by mass is white paint. More white paint is added until the mixture is 90% white paint by mass. What is the resulting total mass of the paint, in kg?
18. In a regular polygon, a diagonal is a line segment joining a vertex to any non-neighbouring vertex. For example, a regular hexagon has 9 diagonals. If a regular polygon with  $n$  sides has 90 diagonals, what is the value of  $n$ ?
19. Determine the number of triples  $(a, b, c)$  of three positive integers with  $a < b < c$  whose sum is 100 and whose product is 18 018.
20. We define  $(a, b) \diamond (c, d) = (ac - bd, ad + bc)$ .  
Determine all pairs  $(x, y)$  for which  $(x, 3) \diamond (x, y) = (6, 0)$ .
21. A *standard die* is a cube whose faces are labelled 1, 2, 3, 4, 5, 6. Raffi rolls a standard die three times obtaining the results  $p, q$  and  $r$  on the first, second and third rolls, respectively. What is the probability that  $p < q < r$ ?
22. Determine the largest possible radius of a circle that is tangent to both the  $x$ -axis and  $y$ -axis, and passes through the point  $(9, 2)$ .  
(A circle in the  $xy$ -plane with centre  $(a, b)$  and radius  $r$  has equation  $(x - a)^2 + (y - b)^2 = r^2$ .)
23. For each positive integer  $n$ , define  $s(n)$  to be the sum of the digits of  $n$ . For example,  $s(2014) = 2 + 0 + 1 + 4 = 7$ . Determine all positive integers  $n$  with  $1000 \leq n \leq 9999$  for which  $\frac{n}{s(n)} = 112$ .
24. In the diagram, cube  $ABCDEFGH$  has edges of length  $a$ . Points  $M, N$  and  $P$  are on edges  $GC, FB$ , and  $EA$ , respectively, so that  $MC = \frac{1}{2}a, NB = \frac{1}{3}a$  and  $PA = \frac{1}{4}a$ . If the plane defined by points  $M, N$  and  $P$  intersects  $HD$  at point  $T$ , determine the length  $DT$ .



25. In trapezoid  $ABCD$ ,  $AD$  is parallel to  $BC$  and  $BC : AD = 5 : 7$ . Point  $F$  lies on  $AD$  and point  $E$  lies on  $DC$  so that  $AF : FD = 4 : 3$  and  $CE : ED = 2 : 3$ . If the area of quadrilateral  $ABEF$  is 123, determine the area of trapezoid  $ABCD$ .

