The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING
cemc.uwaterloo.ca

Canadian Intermediate Mathematics Contest

Thursday, November 20, 2014
(in North America and South America)

Friday, November 21, 2014
(outside of North America and South America)

UNIVERSITY OF WATERLOO

Time: 2 hours

Calculators are allowed, with the following restriction: you may not use a device that has internet access, that can communicate with other devices, or that contains previously stored information. For example, you may not use a smartphone or a tablet.

Do not open this booklet until instructed to do so.
There are two parts to this paper. The questions in each part are arranged roughly in order of increasing difficulty. The early problems in Part B are likely easier than the later problems in Part A.

PART A

1. This part consists of six questions, each worth 5 marks.
2. **Enter the answer in the appropriate box in the answer booklet.**
   For these questions, full marks will be given for a correct answer which is placed in the box. Part marks will be awarded **only if relevant work** is shown in the space provided in the answer booklet.

PART B

1. This part consists of three questions, each worth 10 marks.
2. **Finished solutions must be written in the appropriate location in the answer booklet.** Rough work should be done separately. If you require extra pages for your finished solutions, they will be supplied by your supervising teacher. Insert these pages into your answer booklet. Be sure to write your name, school name and question number on any inserted pages.
3. Marks are awarded for completeness, clarity, and style of presentation. A correct solution, poorly presented, will not earn full marks.

At the completion of the contest, insert your student information form inside your answer booklet.

Do not discuss the problems or solutions from this contest online for the next 48 hours.

The name, grade, school and location, and score range of some top-scoring students will be published on the Web site, cemc.uwaterloo.ca. In addition, the name, grade, school and location, and score of some students may be shared with other mathematical organizations for other recognition opportunities.

Canadian Intermediate Mathematics Contest

NOTE:
1. Please read the instructions on the front cover of this booklet.
2. Write solutions in the answer booklet provided.
3. It is expected that all calculations and answers will be expressed as exact numbers such as 4π, $2+\sqrt{7}$, etc., rather than as $12.566\ldots$ or $4.646\ldots$.
4. While calculators may be used for numerical calculations, other mathematical steps must be shown and justified in your written solutions and specific marks may be allocated for these steps. For example, while your calculator might be able to find the x-intercepts of the graph of an equation like $y = x^3 - x$, you should show the algebraic steps that you used to find these numbers, rather than simply writing these numbers down.
5. Diagrams are not drawn to scale. They are intended as aids only.
6. No student may write both the Canadian Senior Mathematics Contest and the Canadian Intermediate Mathematics Contest in the same year.

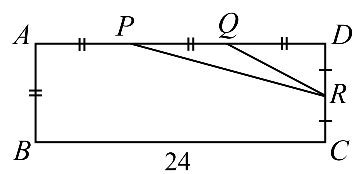
PART A

For each question in Part A, full marks will be given for a correct answer which is placed in the box. Part marks will be awarded only if relevant work is shown in the space provided in the answer booklet.

1. There are 200 people at the beach, and 65% of these people are children. If 40% of the children are swimming, how many children are swimming?

2. If $x + 2y = 14$ and $y = 3$, what is the value of $2x + 3y$?

3. In the diagram, $ABCD$ is a rectangle with points P and Q on AD so that $AB = AP = PQ = QD$. Also, point R is on DC with $DR = RC$. If $BC = 24$, what is the area of $\triangle PQR$?



4. At a given time, the depth of snow in Kingston is 12.1 cm and the depth of snow in Hamilton is 18.6 cm. Over the next thirteen hours, it snows at a constant rate of 2.6 cm per hour in Kingston and at a constant rate of x cm per hour in Hamilton. At the end of these thirteen hours, the depth of snow in Kingston is the same as the depth of snow in Hamilton. What is the value of x?

5. Scott stacks golfballs to make a pyramid. The first layer, or base, of the pyramid is a square of golfballs and rests on a flat table. Each golfball, above the first layer, rests in a pocket formed by four golfballs in the layer below (as shown in Figure 1). Each layer, including the first layer, is completely filled. For example, golfballs can be stacked into a pyramid with 3 levels, as shown in Figure 2. The four triangular faces of the pyramid in Figure 2 include a total of exactly 13 different golfballs. Scott makes a pyramid in which the four triangular faces include a total of exactly 145 different golfballs. How many layers does this pyramid have?

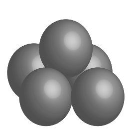


Figure 1

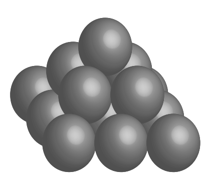


Figure 2

6. A positive integer is a *prime number* if it is greater than 1 and has no positive divisors other than 1 and itself. For example, the number 5 is a prime number because its only two positive divisors are 1 and 5.

   The integer 43797 satisfies the following conditions:

   - each pair of neighbouring digits (read from left to right) forms a two-digit prime number, and
   - all of the prime numbers formed by these pairs are different,

   because 43, 37, 79, and 97 are all different prime numbers. There are many integers with more than five digits that satisfy both of these conditions. What is the largest positive integer that satisfies both of these conditions?

PART B

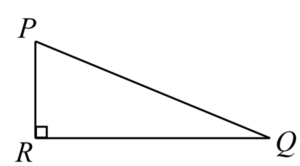
For each question in Part B, your solution must be well organized and contain words of explanation or justification. Marks are awarded for completeness, clarity, and style of presentation. A correct solution, poorly presented, will not earn full marks.

1. (a) Determine the average of the six integers $22, 23, 23, 25, 26, 31$.

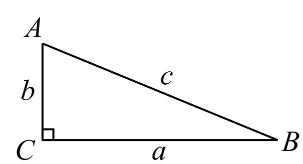
   (b) The average of the three numbers $y + 7, 2y - 9, 8y + 6$ is 27. What is the value of y?

   (c) Four positive integers, not necessarily different and each less than 100, have an average of 94. Determine, with explanation, the minimum possible value for one of these integers.

2. (a) In the diagram, $\triangle PQR$ is right-angled at R. If $PQ = 25$ and $RQ = 24$, determine the perimeter and area of $\triangle PQR$.



   (b) In the diagram, $\triangle ABC$ is right-angled at C with $AB = c$, $AC = b$, and $BC = a$. Also, $\triangle ABC$ has perimeter 144 and area 504. Determine all possible values of c.



   (You may use the facts that, for any numbers x and y, $(x+y)^2 = x^2+2xy+y^2$ and $(x-y)^2 = x^2-2xy+y^2$.)

3. Vicky starts with a list (a, b, c, d) of four digits. Each digit is 0, 1, 2, or 3. Vicky enters the list into a machine to produce a new list (w, x, y, z). In the new list, w is the number of 0s in the original list, while x, y and z are the numbers of 1s, 2s and 3s, respectively, in the original list. For example, if Vicky enters $(1, 3, 0, 1)$, the machine produces $(1, 2, 0, 1)$.

   (a) What does the machine produce when Vicky enters $(2, 3, 3, 0)$?

   (b) Vicky enters (a, b, c, d) and the machine produces the identical list (a, b, c, d). Determine all possible values of $b + 2c + 3d$.

   (c) Determine all possible lists (a, b, c, d) with the property that when Vicky enters (a, b, c, d), the machine produces the identical list (a, b, c, d).

   (d) Vicky buys a new machine into which she can enter a list of ten digits. Each digit is 0, 1, 2, 3, 4, 5, 6, 7, 8, or 9. The machine produces a new list whose entries are, in order, the numbers of 0s, 1s, 2s, 3s, 4s, 5s, 6s, 7s, 8s, and 9s in the original list. Determine all possible lists, L, of ten digits with the property that when Vicky enters L, the machine produces the identical list L.