WATERLOO | MATHEMATICS

The CENTRE for EDUCATION in MATHEMATICS and COMPUTING

2014 Beaver Computing Challenge

Questions

Answers, Explanations, and Connections

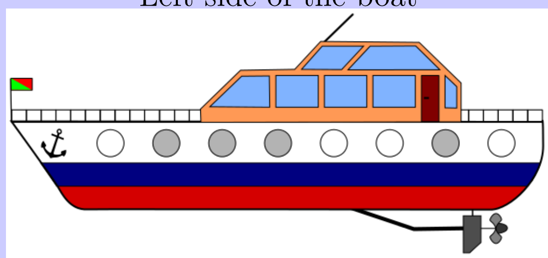
Part A

Funny Windows

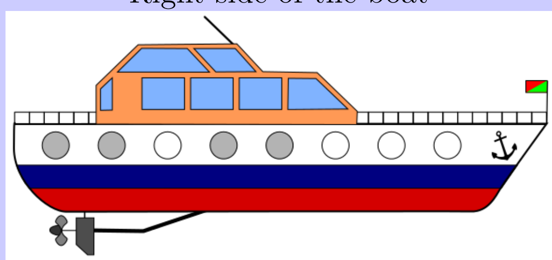
Story

Glass used to make each window on a boat is either clear or lightly tinted. The left side of a boat has eight circular windows. Directly across from these windows are another eight windows on the right side of the boat. The boat is shown below.

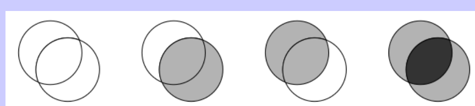
Left side of the boat



Right side of the boat

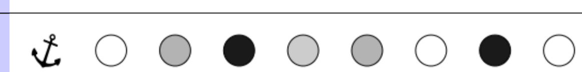


When two pieces of glass overlap, one sees either clear, lightly tinted or darkly tinted glass as shown below.



Question

What colours does one see when standing on land looking straight through corresponding windows of the boat above?

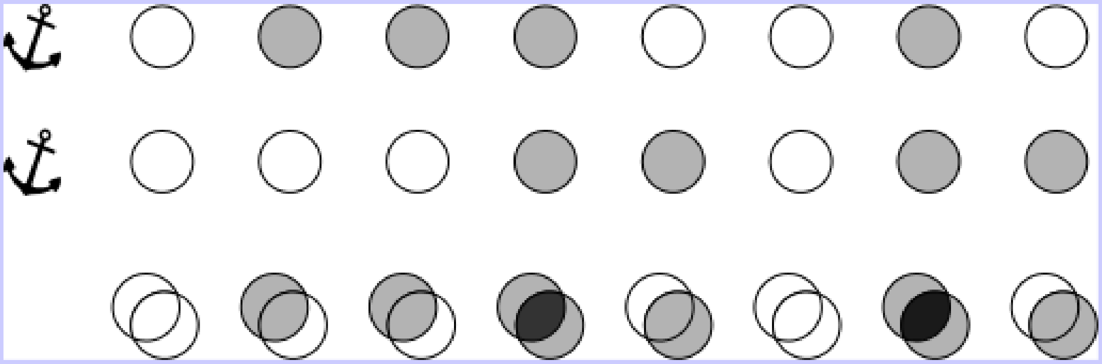
(A)	
(B)	
(C)	
(D)	

Answer

(C) 

Explanation of Answer

Using the anchor as a reference point, we line up corresponding windows as shown below and then use the given information about overlapping glass.



Connections to Computer Science

We usually think of all computer data as a sequence of 0s and 1s called *binary digits*. At the root of any computation are *operators* or *connectives* that consume two binary digits and produce more data. If we equate clear glass to 0 and any tinted glass (both light and dark) to 1, then the above *operation* corresponds to the `OR` *operator*.

Another way to model the story is to use vectors. A *vector* can be used to represent a point in two or three (or even more!) dimensions. Many graphics are represented using *vectors*. Mathematical operations on *vectors* can describe changes such as the animation in a video game. In the story above, if the clear, lightly tinted and darkly tinted glass corresponds to 0, 1 and 2 respectively, then the problem corresponds to adding two vectors: $\langle 0\ 1\ 1\ 1\ 0\ 0\ 1\ 0 \rangle + \langle 0\ 0\ 0\ 1\ 1\ 0\ 1\ 1 \rangle = \langle 0\ 1\ 1\ 2\ 1\ 0\ 2\ 1 \rangle$.

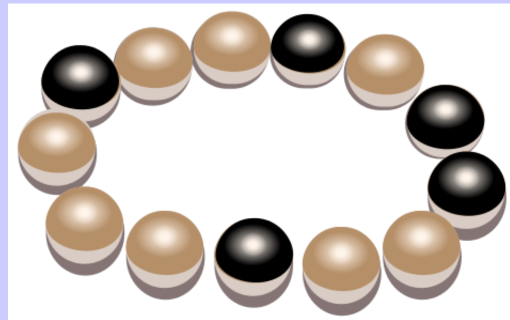
Country

Canada 

Pearl Bracelet

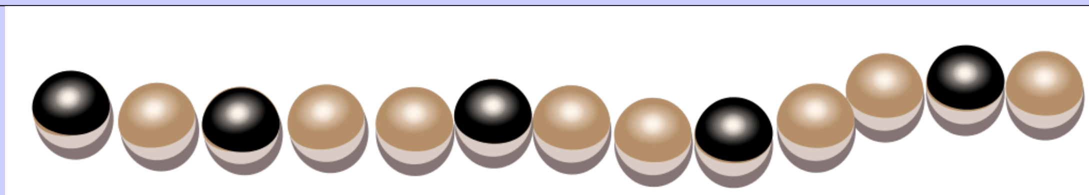
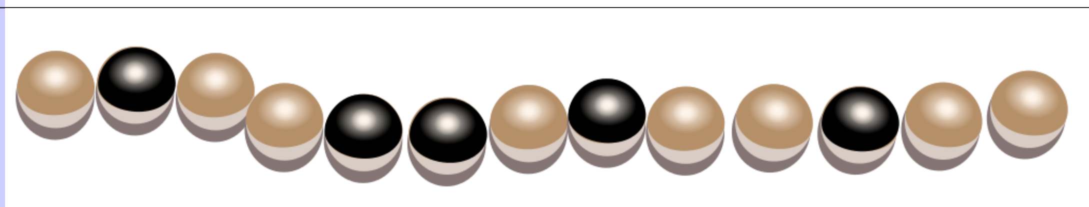
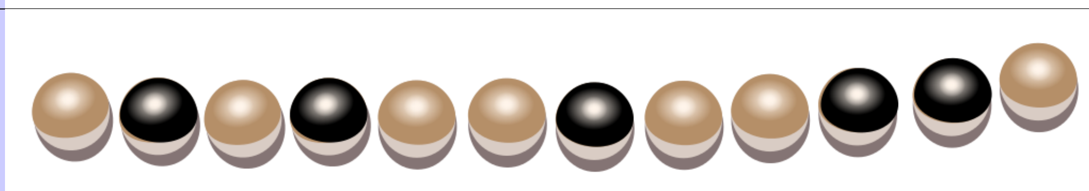
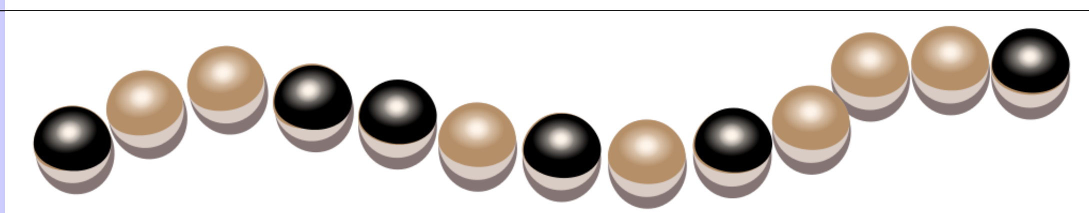
Story

For the grand ball, a princess wore the bracelet with dark and light pearls shown to the right. After the ball, she unfastened the bracelet between two pearls and put it in a chest. The next evening, she wanted to wear the same bracelet but there were many similar bracelets in the chest.



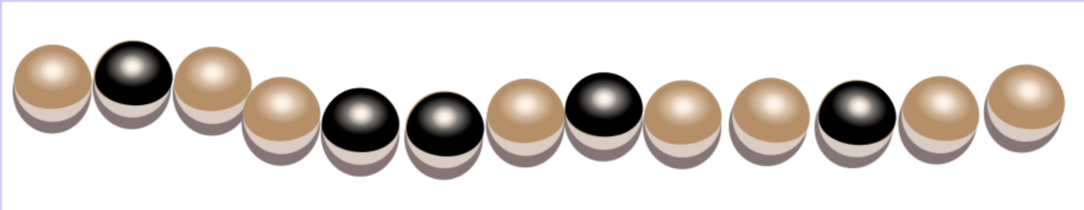
Question

Which of the following bracelets did the princess wear to the grand ball?

(A)	
(B)	
(C)	
(D)	

Answer

(B)



Explanation of Answer

Notice that the correct bracelet has 5 dark and 8 light pearls for a total of 13 pearls. Moreover, two dark pearls are side-by-side on the correct bracelet.

Answer (A) is incorrect because every dark pearl is beside two light pearls.

Answer (C) is incorrect because it has only 12 pearls.

Answer (D) is incorrect because it has 6 dark pearls.

The correct bracelet is formed by attaching the pearls at the end of the bracelet in Answer (B).

Connections to Computer Science

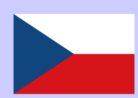
The bracelet is an example of a *sequence* of objects. The pearls are arranged in a certain *pattern*. When identifying the correct bracelet you have to look for properties of this *pattern*.

In informatics, *pattern matching* means finding similar objects in different sources. Geneticists and other medical researchers use *pattern matching* to examine and compare very long strings of human DNA to better understand diseases in the search for cures and treatments.

Other uses of *pattern matching* include locating a small image within a bigger one (*image processing*) and searching for a word in a large amount of text (*text processing*).

Country

Czech Republic



Broken Clock

Story

A digital clock displays four digits. Each digit is displayed using seven segments that are each either on or off as shown below.

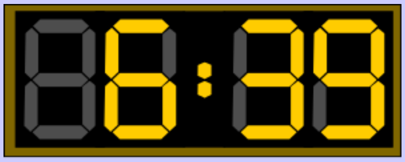
The clock breaks. Exactly one of the seven segments of one digit does not turn on.

Question

If the broken clock displays the time above, which of the following might be the real time?

(A)	
(B)	
(C)	
(D)	

Answer

(D) 

Explanation of Answer

If one of the segments of a seven-segment display does not work, the possible corrections for the digits 6, 3 and 5 are:

Therefore, possible correct "times" are 8:35, 6:95, 6:36 and 6:39.

8:35 is not included in the alternatives.

6:95 does not represent a valid time.

6:36 is not included in the alternatives.

The real time must be 6:39.

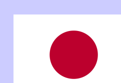
Connections to Computer Science

Computers are generally very reliable machines. However, unexpected events, such as power failures, do occur. Sometimes this introduces errors. *Error-correcting* methods are used to try and detect when this happens and fix mistakes when possible. These techniques are usually not perfect but any remaining errors are not always noticeable to humans. For example, a very small change to an audio file or minor edit to a video may not be noticeable to the human ear or eye.

The digital clocks in this story use seven-segment displays. This is an *internal representation* of numerals. Computer scientists need to balance how data is viewed, how it is stored and the many forms it takes in-between these two *levels of abstraction*.

Country

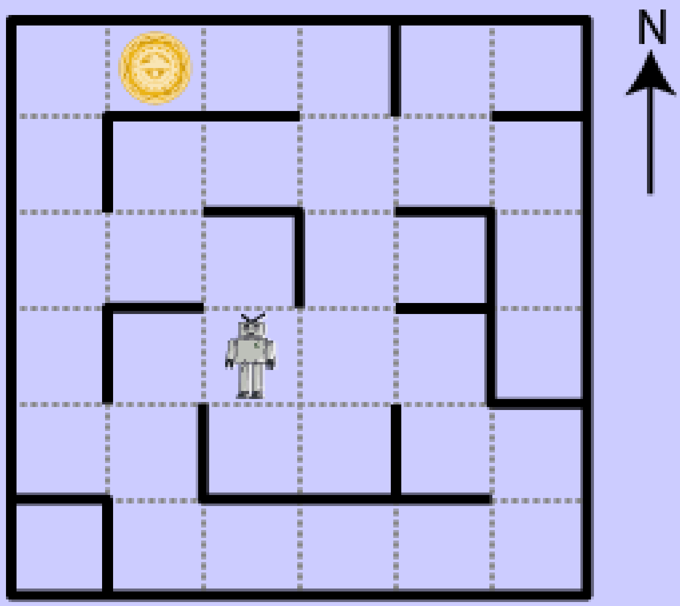
Japan



Alien Language

Story

Space beavers are on a mission to find the artifact hidden in the 6-by-6 maze below. Their robot discovered four sequences of alien words. Now, they must program their robot to follow one of these sequences and end on the same space as the artifact. They know the aliens use a different word for each of "move one space North", "move one space South", "move one space East" and "move one space West".



Question

If the robot and artifact are located as shown above, one of the four sequences below is correct. Which is it?

(A) Ha', Ha', poS, Ha'

(B) Ha', poS, poS, Ha', nIH, Ha'

(C) Ha', poS, poS, Ha', Ha', nIH

(D) Ha', poS, nIH, vI'ogh, Ha', poS

Answer

(C) Ha’, poS, poS, Ha’, Ha’, nIH

Explanation of Answer

Answer (A) is too short: it is impossible to reach the artifact in fewer than six steps.

Answer (B) cannot be correct. If Ha’ means West, then poS could be South or East. If poS is South, the robot bumps into a wall on the fourth move; if poS is East, then after 4 moves, the robot is at his original position, and cannot reach the artifact in the remaining two steps. If Ha’ is East, then poS must be North, but the robot moves away in the fourth move and not enough moves remain to bring the robot to the artifact. If Ha’ is North, then poS could be South (and the robot would end up back at the original position after four moves) or poS could be West, which would force nIH to be South, and we would not reach the artifact. If Ha’ is South, then poS must be North, and the robot would be at the original position after four moves.

The first four steps of Answer (D) move the robot in each direction once (in an unknown order), thus after the first four commands the robot is back where it started. It cannot reach the artifact in the remaining two steps.

The robot can each the artifact by moving North, West, West, North, North and then East. This corresponds to Answer (C) where Ha’ means North, poS means West and nIH means East.
Another way to notice this solution is to observe that the last three moves of the robot will be of the form NNE or NWW, where we do not know the words for North (N), East (E) or West (W). Only answer (C) has the last three moves of this form.

Connections to Computer Science

Cryptanalysis is the science of reading hidden messages. Since ancient times, experts called *cryptanalysts* have been deciphering messages sent by enemies. In doing so, they use their knowledge about the words that might possibly form the hidden message. For example, in reading the messages enciphered by the famous Enigma machine during the Second World War, British *cryptanalysts* searched for the names of German cities and words related to weather reports.

Country

Slovenia

Truth

Story

Beaver Bob only tells the truth on Monday, Wednesday and Friday and always lies on all other days of the week. Today he says: “Tomorrow I will tell the truth.”

Question

What day is it?

(A) Tuesday

(B) Friday

(C) Saturday

(D) Sunday

Answer

(C) Saturday

Explanation of Answer

If Bob's statement was the truth, this would mean that he tells the truth two days in a row, which never happens. Therefore, Bob's statement was a lie which implies that he will lie tomorrow. This means it must be Saturday, since Saturday and Sunday are the only two days in a row when Bob lies.

Connections to Computer Science

Logic is fundamental to computer science. When designing computer programs, careful thought must go into logical structures. Complex computations can be made much more efficient if careful logic is used to simplify the situation. For example, instead of trying every possibility, one can exclude unnecessary computations, such as trying all weekdays in the story above.

Logic is very important to *mathematics* and *mathematicians*. It is one of many ways that computer science and *mathematics* are closely related disciplines that strongly depend on each other.

Country

Hungary

Part B

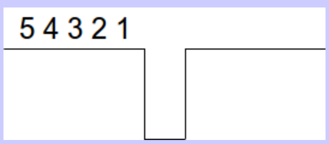
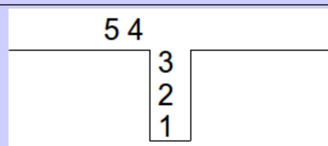
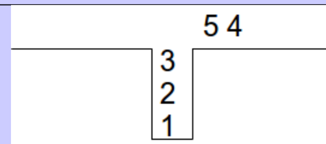
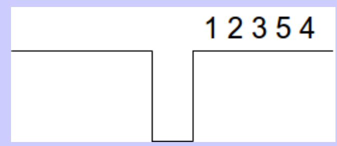
Rabbit Hole

Story

Beavers are going for a stroll in the woods. They walk in a line, one beaver after another.

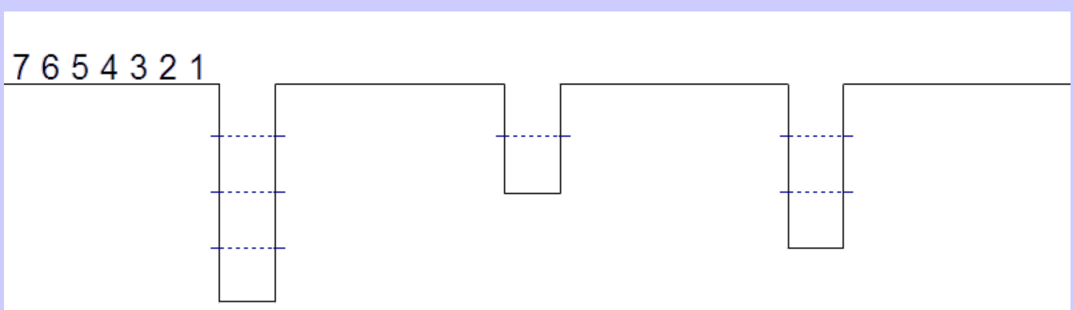
Nasty rabbits have dug holes along the beavers' route.

The holes are deep enough so that some number of beavers will fall in. Once the hole is full of beavers, all the beavers behind the hole walk on top of the beavers in the hole. Then the beavers climb out of the hole from the top to the bottom. The example below involves beavers 1, 2, 3, 4, 5; (1 being the first one in line, and 5 being the last in line) and one hole deep enough for three beavers.

Initially	First three beavers all in the hole	Walking on top of beavers in the hole	Every beaver out of the hole and back in line
			

Question

If there are 7 beavers (with 1 being the first one in line, and 7 being the last in line), and the first hole encountered holds four beavers, the second hole encountered holds two beavers and the last hole encountered hold three beavers, what is the order of the beavers after all beavers have passed over these three holes?



(A) 3 2 1 6 5 7 4

(B) 7 4 3 5 6 1 2

(C) 1 2 3 4 7 5 6

(D) 2 3 4 1 6 7 5

Answer

(B) 7 4 3 5 6 1 2

Explanation of Answer

Initially, the line is:

7 6 5 4 3 2 1

Then, after the first hole (of depth 4), we have:

1 2 3 4 7 6 5

After the second hole (of depth 2), we have:

5 6 1 2 3 4 7

After the third hole (of depth 3), we have:

7 4 3 5 6 1 2

Another way to see the solution is to notice that the first two holes encountered have 6 spaces, and thus, we the line approaches the last hole, “7” will be at the front, and thus, “7” will be the last to leave the last hole. Thus, the answer must be (B).

Connections to Computer Science

Organizing data in a structured way is important, and there are many different *data structures* that can be used for this purpose. This task shows an example of a structure called a *stack*, which works similarly to stacking plates on top of each other. New plates are added on top of the *stack* and have to be removed from the top one at a time. This type of structure is commonly referred to as a *LIFO-structure*, the objects that have been added last are the first to be removed (Last-In First-Out).

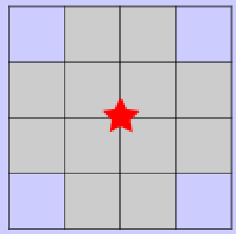
Country

Canada

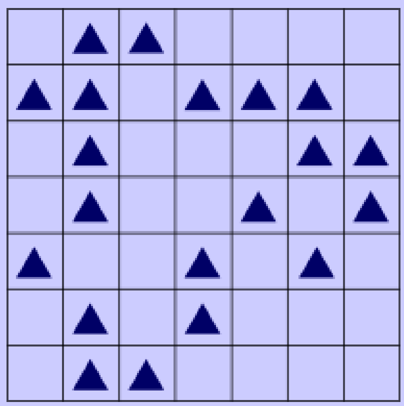
Loudspeakers in a Village

Story

Loudspeakers are set up to announce information to villagers. Each speaker must be located at a point where two grid lines cross. As illustrated below, sound from each speaker reaches twelve grey squares.



The figure below is a map of the village. Triangles represent the locations of houses. It must be possible to hear information from at least one speaker from each house.



Question

What is the fewest number of speakers needed?

(A) 2

(B) 3

(C) 4

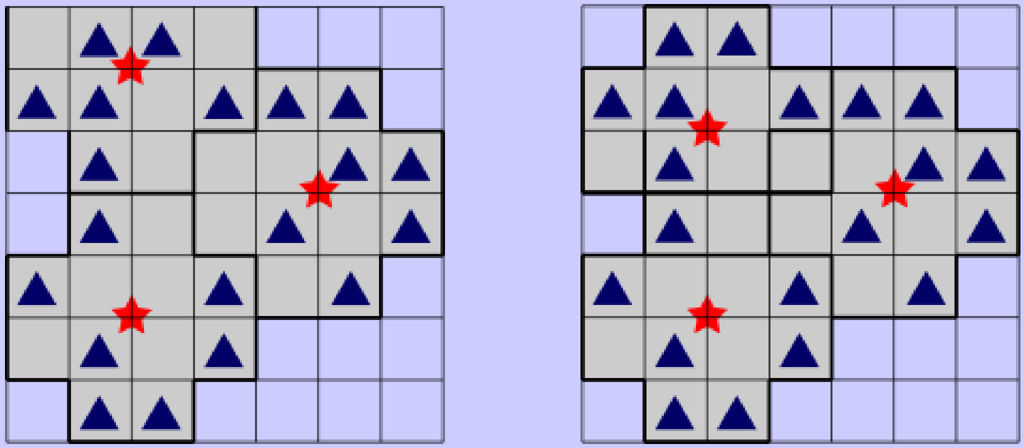
(D) 5

Answer

(B) 3

Explanation of Answer

Either of the following two arrangements of three speakers covers all of the houses.



You cannot cover all of the houses with two speakers. To see this, notice that a speaker that covers either of the rightmost houses, cannot cover the topmost or bottommost houses. Also, no speaker can cover both the topmost and bottommost houses. Therefore, at least three speakers are needed.

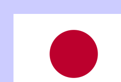
Connections to Computer Science

Similar to dividing space into a number of regions, *covering* space with figures is useful in a number of applications. One example arises when testing programs. To ensure that a program is correct, it is wise to ensure input data tests (*covers*) every line of code. Other examples are noted with the connections listed for the Robust Network task later in this document.

In this task, you are required to find the *optimal* way to *cover* the region. It is often very difficult to design an algorithm that will always find a maximum or minimum. When no algorithm is known or all known algorithms are too slow, we often resort to *randomized algorithms* and *approximation algorithms* which are usually fast enough and usually "close" to optimal.

Country

Japan



Phone Bills

Story

A communications company stores billing information. There are exactly three charges for each customer (for data, voice and text). Each customer has his or her own unique and distinct phone number. There are two storage options:

OPTION A

Store all the information in one table. Each row in the table corresponds to a data charge, voice charge or text charge.

NAME	PHONE NUMBER	CHARGE
Aki	458-6578	10.00
Aki	458-6578	15.00
Aki	458-6578	10.00
Vlad	235-8998	40.00
Vlad	235-8998	40.00
Vlad	235-8998	30.00
Mia	515-6632	25.00
Mia	515-6632	20.00
Mia	515-6632	20.00

OPTION B

Store the phone number for each customer in one table. Store the charges in a second table in which each row corresponds to a data charge, voice charge or text charge.

NAME	PHONE NUMBER
Aki	458-6578
Vlad	235-8998
Mia	515-6632

PHONE NUMBER	CHARGE
458-6578	10.00
458-6578	15.00
458-6578	10.00
235-8998	40.00
235-8998	40.00
235-8998	30.00
515-6632	25.00
515-6632	20.00
515-6632	20.00

The amount of storage is measured in bytes. Each name requires 128 bytes. Each phone number requires 4 bytes. Each charge requires 4 bytes. These measurements do not depend on how long names are or how big charges are.

Question

Suppose A and B are the amounts of storage in bytes required by options **A** and **B** respectively. If the company has 1000 cell phone customers, which of the following statements is true?

(A) $A = B$

(B) $A < B$

(C) $A > B$ and $A < 2B$

(D) $A \geq 2B$

Answer

(D) $A >= 2B$

Explanation of Answer

Option A uses $128 + 4 + 4 = 136$ bytes per row. There are 3 rows per customer giving a total of 3000 rows. Therefore $A = 136 \times 3000 = 408000$.

Option B uses $4 + 128 = 132$ bytes per row in the first table which has 1000 rows. It uses $4 + 4 = 8$ bytes per row in the second table with 3000 rows. Therefore $B = 132000 + 8 \times 3000 = 156000$ bytes.

Since $2 \times 156 < 408$, then $A \geq 2B$.

Connections to Computer Science

A *database* stores a large amount of information for a long period of time. This data is often placed in *tables*. Expertise is required to determine the number of tables and which information is placed in which table. Different designs must try to balance the amount of memory used with the amount of time that will be needed to answer questions (called *queries*) about the data.

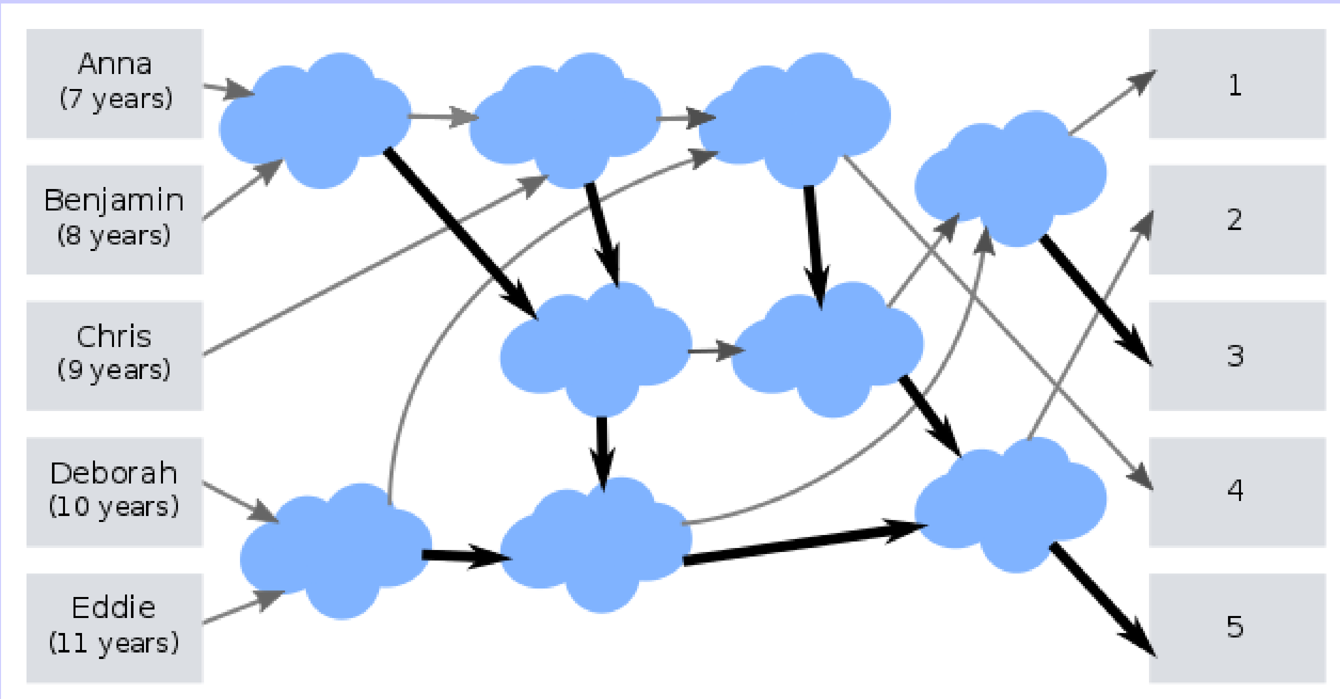
Country

Canada

Network Game

Story

Five beavers, Anna (age 7), Benjamin (age 8), Chris (age 9), Deborah (age 10) and Eddie (age 11) are playing a game where they walk through the clouds following arrows shown below. At every cloud, they wait for another beaver to arrive. Then the older beaver leaves along the thick arrow, while the younger one leaves along the thin arrow.



Question

The exits are numbered 1, 2, 3, 4 and 5. Which exit does each beaver reach at the end of the game?

(A)	(B)	(C)	(D)
1: Anna	1: Eddie	1: Benjamin	1: Benjamin
2: Benjamin	2: Deborah	2: Deborah	2: Chris
3: Chris	3: Chris	3: Chris	3: Deborah
4: Deborah	4: Benjamin	4: Anna	4: Anna
5: Eddie	5: Anna	5: Eddie	5: Eddie

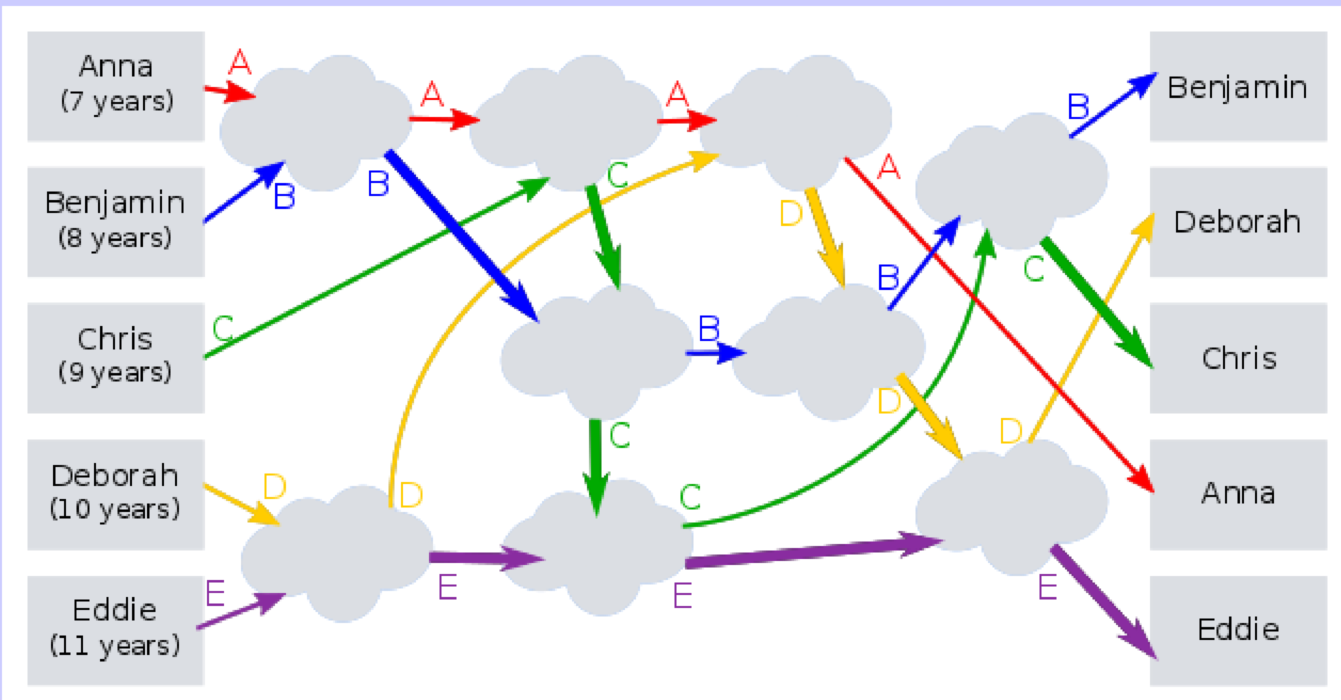
Answer

(C)

1: Benjamin
2: Deborah
3: Chris
4: Anna
5: Eddie

Explanation of Answer

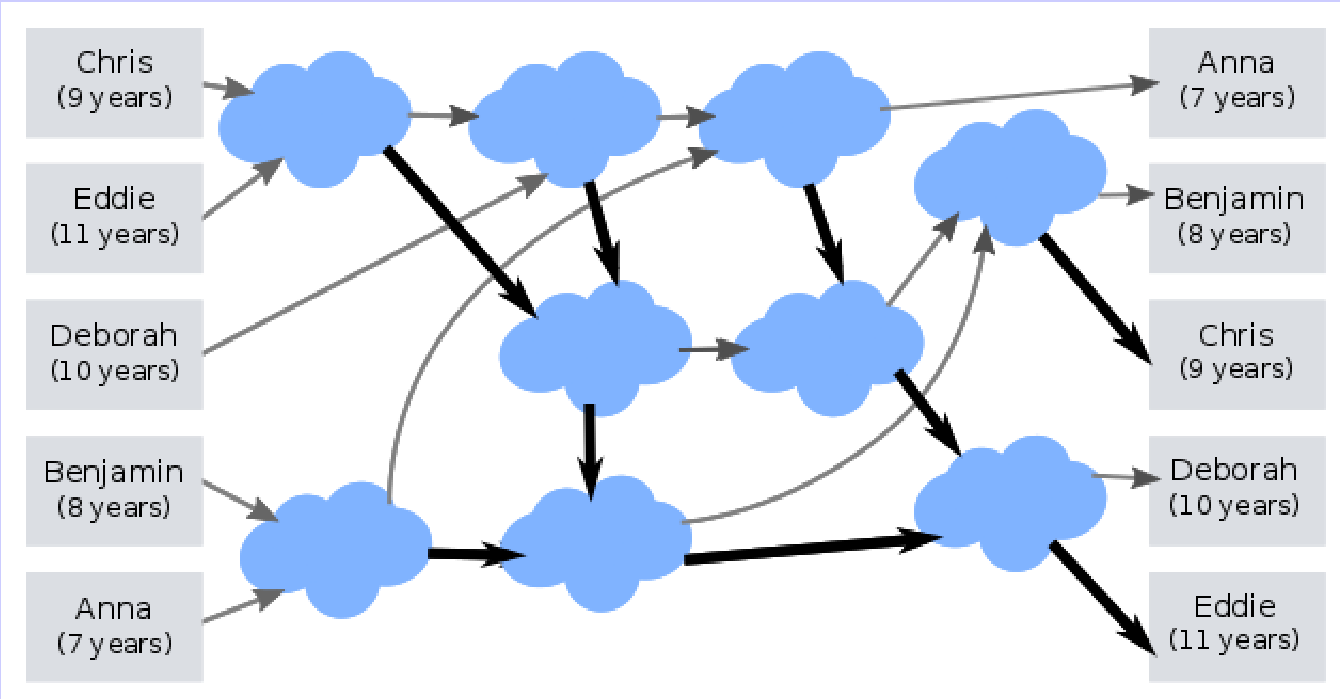
Answer (C) is correct. The beavers flow through the clouds as shown below.



We can see that Answers (A) and (B) are wrong relatively quickly by noting that the youngest beaver Anna always follows thin arrows leading her to the 4th exit, and the oldest beaver Eddie always follows thick arrows leading him to the 5th exit. We need to follow the flow of beavers shown above to see that Answer (D) is incorrect.

Connections to Computer Science

The game is played on a *network*. The arrows correspond to *input values* and *output values*. The clouds correspond to *comparisons*. If this network is designed in a certain way (for example, as shown below), it can *sort* a sequence of values. Such a network is called a *sorting network*.



Country

Switzerland

Lazy Beaver

Story

A lazy beaver hires five strong beavers. Everyday, each working beaver receives orders to either collect logs from the forest and bring them to the warehouse, or remove logs from the warehouse for processing. Initially, the warehouse has 100 logs. The number of logs added or removed for each beaver is given in the table below.

	Collect	Process
Beaver A	Add 81 logs to warehouse	Remove 81 logs from warehouse
Beaver B	Add 27 logs to warehouse	Remove 27 logs from warehouse
Beaver C	Add 9 logs to warehouse	Remove 9 logs from warehouse
Beaver D	Add 3 logs to warehouse	Remove 3 logs from warehouse
Beaver E	Add 1 log to warehouse	Remove 1 log from warehouse

If a beaver is on vacation, it does not add or remove logs from the warehouse. For example, if Beavers A and D are on vacation, Beaver B is ordered to "Collect" and Beavers C and E are ordered to "Process", then at the end of the day, the warehouse will have $100 + 27 - 9 - 1 = 117$ logs.

Question

Which of the following orders will leave 168 logs at the end of a day?

(A) Beavers A, D, and E "Collect"; Beavers B and C "Process".

(B) Beavers A and E "Collect"; Beavers B and D "Process"; Beaver C is on vacation.

(C) Beavers A and B "Collect"; Beavers D, E "Process"; Beaver C is on vacation.

(D) Beaver A "Collect"; Beavers C, D and E "Process"; Beaver B is on vacation.

Answer

(D) Beaver A "Collect"; Beavers C, D and E "Process"; Beaver B is on vacation.

Explanation of Answer

Since $100 + 81 + 0 - 9 - 3 - 1 = 168$, then Beavers A, B, C, D and E work order can Collect, Vacation, Process, Process, Process, respectively. In fact, this must be what happens because, although we don't prove it here, it is the unique way to yield 168 logs at the end of a day.

To verify that the other answers are incorrect, we can calculate:

$100 + 81 - 27 - 9 + 3 + 1 = 149 \neq 168$.

$100 + 81 - 27 + 0 - 3 + 1 = 152 \neq 168$.

$100 + 81 + 27 + 0 - 3 - 1 = 204 \neq 168$.

Connections to Computer Science

Some early computers were based on a *ternary* system. Instead of machines being based on a *binary* system (0 or 1; true or false; on or off) like computers today, these older computers were based on three possible values. They had some computational advantages, including lower power consumption and faster arithmetic operations involving negative values.

Country

Canada

Part C

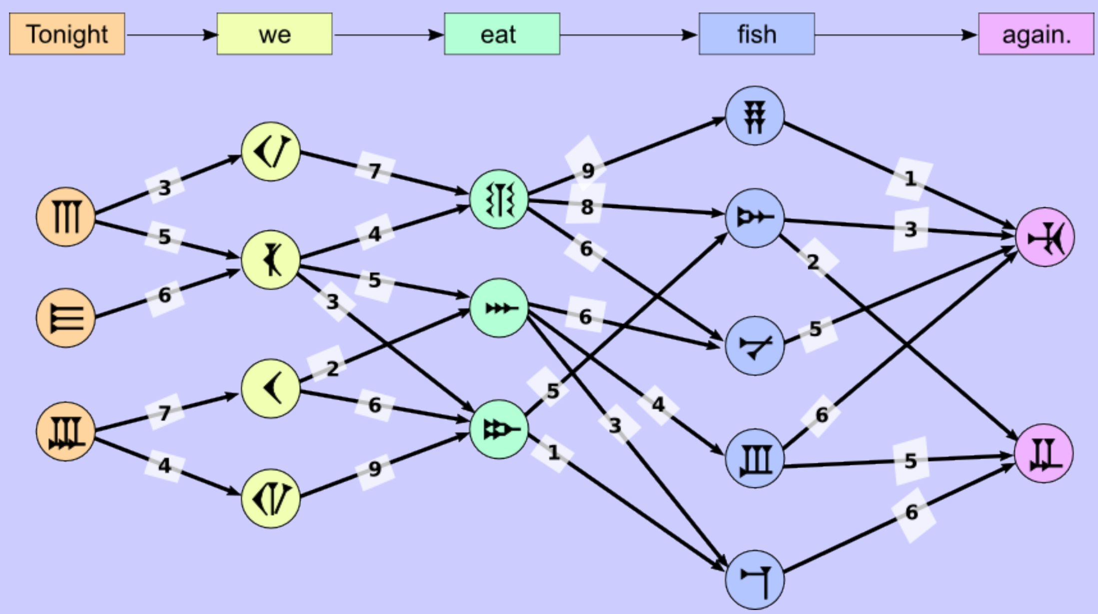
Translation Machine

Story

Betty programs a machine that translates an English sentence to a Beavarian sentence one word at a time. However, there are several possible Beavarian words for each English word!

Betty noticed that different words occur next to each other at different rates. For example, "smart beaver" is more common than "intelligent beaver." She gives scores for word pairs: the higher the score, the more common the word pair is.

An English sentence with five words must be translated into five Beavarian symbols. In the picture below, arrows labelled with scores connect all valid word pairs. The total score for a translation is the sum of the scores of the four arrows used.



Question

What is the highest possible total score for a translation of this sentence?

(A) 18

(B) 21

(C) 22

(D) 23

Answer

(C) 22

Explanation of Answer

The best possible score is 22. This can be achieved by choosing the second, second, second, third and finally first symbol (counting from the top and moving from left-to-right). How can we find this?

We systematically go word by word from left to right assigning a value to each symbol that is the best possible score for a sentence ending with that symbol.

To begin, we assign a value of 0 to the three possible translations for "Tonight". Then we consider the four symbols for "we". We set the value of each corresponding translation of "Tonight we" to be the maximum label of an arrow coming into the corresponding symbol of "we". Reading from top to bottom, this gives us values of 3, 6, 7, 4 for the values of translations of "Tonight we".

To assign values to the translations for "Tonight we eat", we consider the three symbols for "eat" and take the maximum value obtained by adding the value for "Tonight we" to the maximum label on an arrow into the corresponding symbol for "eat". Reading from top to bottom, this gives us values of 10, 11 and 13 for translations of "Tonight we eat".

Continuing in this way, we end with a best possible score for the full sentence.

Using this method to find the best score, we only use $7 + 8 + 7 = 22$ additions and some comparisons instead of $(19 + 17) \times 3 = 108$ additions if we checked every possible path through all the nodes. Can you see where these numbers come from?

Connections to Computer Science

The algorithmic idea to solve this problem quickly is called *dynamic programming*. It is based on a general idea of systematically building the solution from small chunks to bigger and bigger pieces. If you remember (or write down) the partial results, these partial results can be used to calculate a solution without having to recompute these partial results.

This problem also gives you a glimpse of contemporary *machine translation*. It may be somewhat surprising, but machine translation does not depend on a deep understanding of grammar rules. Rather, it works with enormous databases of texts in different languages, and simply put, looks for good matches, especially with *digrams* and *trigrams* (pairs or triplets of words that occur frequently).

Country

Czech Republic

Strange Words

Story

Beavers consider words containing only the letters a, b, c. There are three different operations we can apply to such words:

- Operation 1: Replace every a with the sequence aa.
- Operation 2: Replace some b with c.
- Operation 3: Insert the letter c anywhere in the word.

We can use any of these operations in any order, and may use an operation many times. For example, if we have the word $abbc$ then using Operation 2 we could get $acbc$, then using Operation 1 we could get $aacbc$, and finally using Operation 1 we could get $aaaacbc$.

Question

Which of the following words is impossible to get if we start with $aabbbbaabbccbbabbc$?

(A) $aaaabbcbaaaabbcccbbaabbc$

(B) $accabcbcaabbccbbaccc$

(C) $aaaabccbaaaabbccbaabcc$

(D) $accccaaaccccaacaabbccbbaabbc$

Answer

(C) *aaaabccbaaaabbccbaabcc*

Explanation of Answer

Answer (A) can be produced by:

$$\begin{aligned} aabbbbaabbccbbabbc &\xrightarrow{1} a\boldsymbol{a}a\boldsymbol{a}bbbba\boldsymbol{a}a\boldsymbol{a}bbccbba\boldsymbol{a}bbc \\ &\xrightarrow{2} aaaabb\boldsymbol{c}baaaabbccbbaabbc \\ &\xrightarrow{3} aaaabbcbaaaabb\boldsymbol{c}ccbbaabbc \end{aligned}$$

Answer (B) can be produced by:

$$\begin{aligned} aabbbbaabbccbbabbc &\xrightarrow{3} a\boldsymbol{c}abbbbaabbccbbabbc \\ &\xrightarrow{3} ac\boldsymbol{c}abbbbaabbccbbabbc \\ &\xrightarrow{2} accab\boldsymbol{c}bbaabbccbbabbc \\ &\xrightarrow{2} accabcb\boldsymbol{c}aabbccbbabbc \\ &\xrightarrow{2} accabcbcaabbccbba\boldsymbol{c}bc \\ &\xrightarrow{2} accabcbcaabbccbbac\boldsymbol{c}c \end{aligned}$$

Answer (D) can be produced by:

$$\begin{aligned} aabbbbaabbccbbabbc &\xrightarrow{1} a\boldsymbol{a}a\boldsymbol{a}bbbba\boldsymbol{a}a\boldsymbol{a}bbccbba\boldsymbol{a}bbc \\ &\xrightarrow{3} a\boldsymbol{c}aaabbbbaaaabbccbbaabbc \\ &\xrightarrow{3} ac\boldsymbol{c}aaabbbbaaaabbccbbaabbc \\ &\xrightarrow{3} acc\boldsymbol{c}aaabbbbaaaabbccbbaabbc \\ &\xrightarrow{3} accc\boldsymbol{c}aaabbbbaaaabbccbbaabbc \\ &\xrightarrow{2} accccaaa\boldsymbol{c}bbbaaaabbccbbaabbc \\ &\xrightarrow{2} accccaaac\boldsymbol{c}bbaaaabbccbbaabbc \\ &\xrightarrow{2} accccaaacc\boldsymbol{c}baaaabbccbbaabbc \\ &\xrightarrow{2} accccaaaccc\boldsymbol{c}aaaabbccbbaabbc \\ &\xrightarrow{3} accccaaaccccaa\boldsymbol{c}aabbccbbaabbc \end{aligned}$$

Answer (C) is impossible: to understand why, it is useful to think about the properties that are preserved by each instruction. These properties are commonly called *invariants*.

- Operation 1 guarantees that the number of a's cannot decrease;
- Operation 2 guarantees that the total number of b's and c's remains the same;
- Operation 3 only increases the number c's.

The original word has 13 letters that are either b or c. In Answer (C), only 12 of the letters are b or c, thus it is impossible to produce this by applying the given instructions, since the total number of b's and c's cannot decrease.

Connections to Computer Science

Programs usually consist of statements. Since programming languages must be clear and concise, the form of statements is described by strict sets of rules similar to those above. This form together with a definition of the *meaning* of each statement gives us a fully defined a programming language.

The original word(s) and rules concerning replacement and alteration of symbols form a *grammar*, and the set of words that are described form the *language generated* by that grammar. The three applications of rules in a certain order used above in the explanation are *derivations* of words. It is worth noting in this example that there are an infinite number of words starting from any word, even though we have just three rules in this grammar.

When we try to reason about programs, it is common to consider *invariants* – properties maintained by some operation(s) – as done in our explanation of the correct answer.

Country

Poland

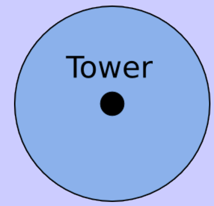
Robust Network

Story

The Beaver TeleCompany wants to place cellphone towers on Windy Island.

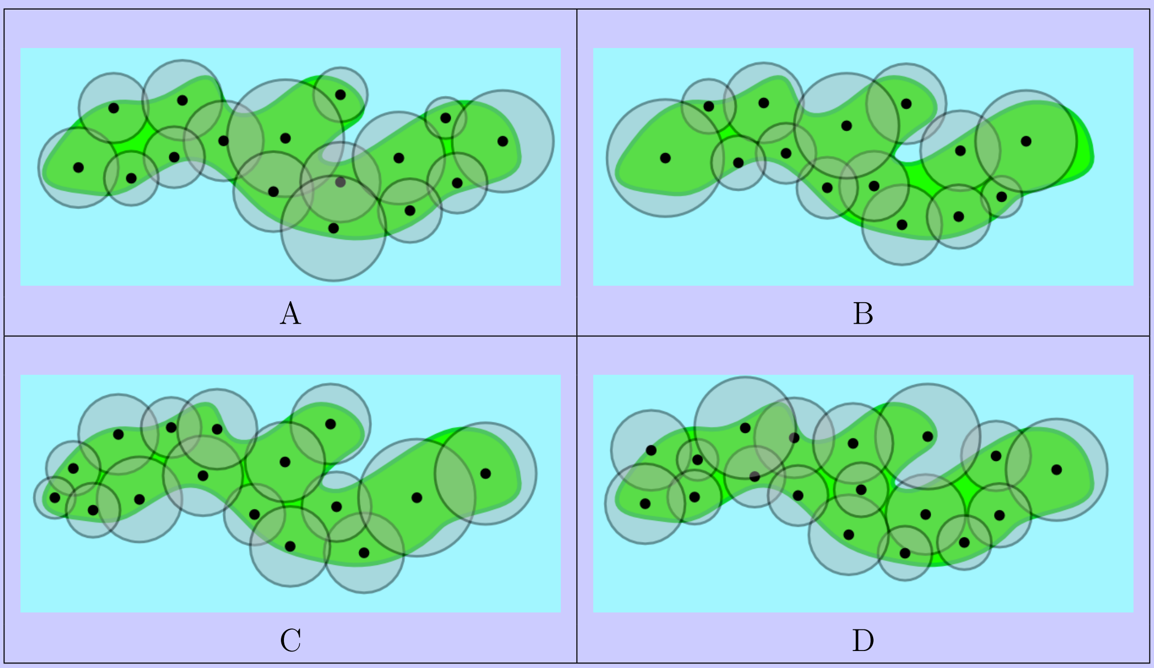
A cellphone tower's coverage area is a circle centered around it. Two towers are connected if their coverage areas overlap. Furthermore, two towers can communicate through a sequence of towers where consecutive towers are connected.

The wind on the island often breaks towers. With any single tower broken, it must be possible for any two of the remaining towers to communicate.

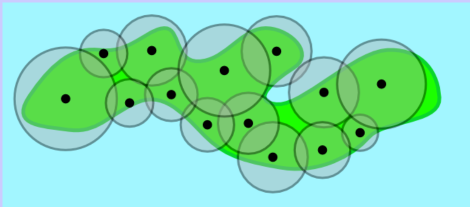


Question

Of the choices below, how should the towers be placed?

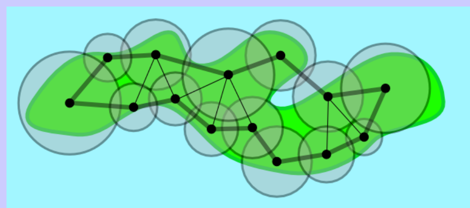


Answer

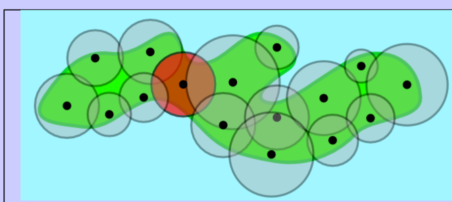
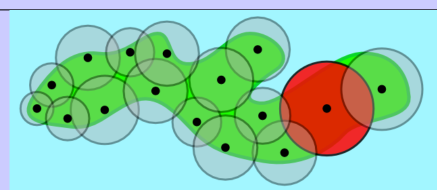
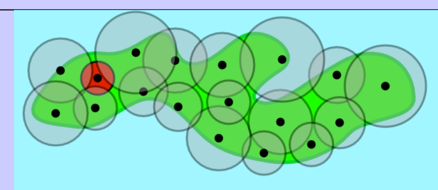
(B) 

Explanation of Answer

The correct answer is B. Following the coast, one can see that the towers are connected in a loop, if a tower breaks, a signal can still be sent between any two remaining towers.



In the other cases, there exists a tower that, if broken, will mean a signal cannot be sent between some pair of towers. One example in each case is highlighted in the pictures below:

		
A	C	D

Connections to Computer Science

The placement of the towers (called *vertices* or *nodes*) and the way they are connected (by *edges*) is called a *graph* or *network topology*. Applications such as cellphone networks, electrical power grids, and transportation networks involve *graphs* in the design of systems that are as reliable as possible. Similar structures can be physical or logical and they can take various shapes (e.g. ring, tree, mesh). We might then also ask other questions and evaluate properties related to different aspects of usability, such as the shortest or longest distance between any two *nodes* in a *graph*.

Country

Hungary 

Height Game

Story

Young beavers Amy, Beavy, Cuttree, Diggy, and Eary, are all different heights. They line up, one after another, facing the same way, in some order. Then each beaver finds all the other beavers that are taller than himself/herself. He/she counts how many of these taller beavers are in front of him/her and how many are behind him/her. The results are shown in the following table:

	Number of taller beavers	
Name	in front	behind
Amy	1	2
Beavy	3	1
Cuttree	1	0
Diggy	0	0
Eary	2	0

Question

In what order are they standing in, from the front of the line to the back of the line?

(A) Diggy, Cuttree, Amy, Beavy, Eary

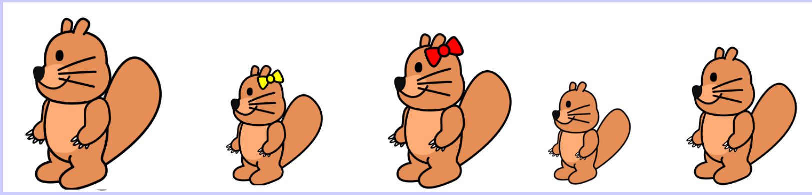
(B) Diggy, Amy, Cuttree, Beavy, Eary

(C) Amy, Cuttree, Diggy, Eary, Beavy

(D) Diggy, Amy, Eary, Beavy, Cuttree

Answer

(B)



Diggy Amy Cuttree Beavy Eary

Explanation of Answer

Since each beaver except Diggy has at least one beaver in front of them, Diggy must be in the first position. Since Diggy also has no taller beavers behind, then Diggy is the tallest beaver. Since Beavy has at least three beavers in front and at least one behind and there are only five beavers, then she must be in the fourth position. This also means Beavy is the shortest beaver.

Diggy (5)			Beavy (1)	

Because Amy has two taller beavers behind her (who must be Cuttree and Eary) and Beavy is shorter than her, Amy must be in the second position.

Diggy (5)	Amy (2)		Beavy (1)	

Finally, since Eary has no taller beavers behind her, she must be behind Cuttree and thus in the last position.

Diggy (5)	Amy (2)	Cuttree (4)	Beavy (1)	Eary (3)

Connections to Computer Science

This problem involves ordering five values based on certain criteria. The most essential ordering concept in computer science is *sorting*. Solutions to many problems may require *sorting* as a necessary first step. It allows us order unordered data and simplify the next steps of an algorithm.

This problem also relies on *logical reasoning*. *Logic* and computer science are deeply connected. When solving a logical problem, as well as writing a computer program, it really helps to approach the problem step by step and establish intermediate results that can then be used to solve the full problem.

Country

Sweden

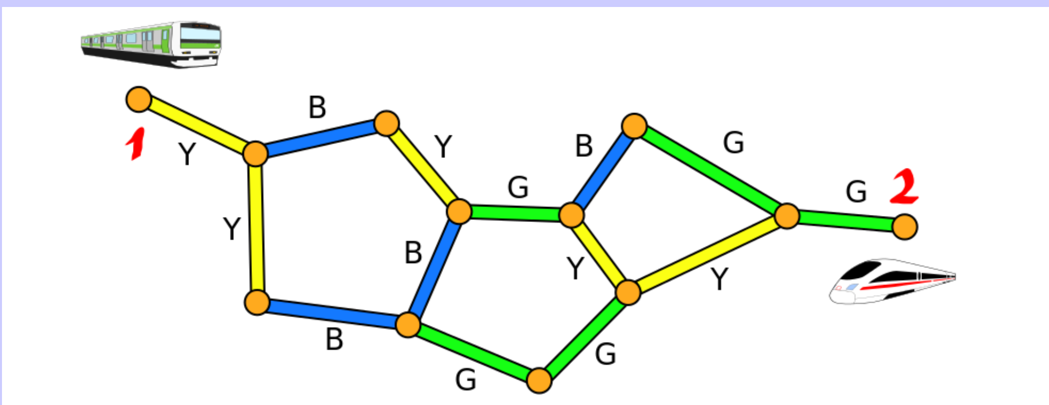
Express Trains

Story

Two trains are going towards each other starting from stations 1 and 2. The map below shows all the stations and coloured rail tracks between them.

At every moment, one of the trains is moving, and the other train is stopped at some station. While a train moves, the colour of the rail track (either Blue, Green or Yellow, marked as B, G or Y, respectively) it is using is recorded. Unfortunately, the record does not store which of the trains was moving.

For example, the record BG can either mean that one of the trains passed over a Blue and then a Green rail track, or it could mean that one train passed over a Blue rail track, and then the other train passed over a Green rail track.



Question

After some number of moves, the two trains meet. One of the following records the trains' movements up to the moment of meeting. Which is it?

(A) GYGBGYBB

(B) YYBYGGBG

(C) GBYBYGY

(D) YBBYBYY

Answer

(A) GYGBGYBB

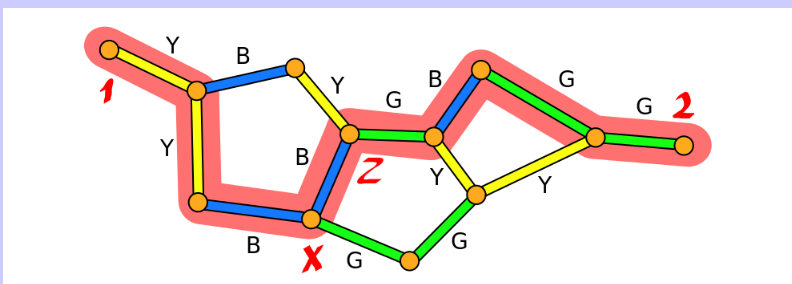
Explanation of Answer

Answer (D) YBBYBYY is incorrect because it does not contain green track G. If you want to leave or enter station 2 you must use the G track.

Answer (C) GBYBYGY is incorrect too. It starts with letter G so the train from station 2 must start moving first. The second letter is B but at that moment, no train is at a station connected to a B track.

Answer (B) YYBYGGBG is incorrect. The YYB at the start of the record must correspond to train 1 because train 2 must start with a G track. The next letter is Y but at that moment, no train is at a station connected to a B track.

Answer (A) GYGBGYBB is correct. Vertical lines in the record G|Y|GBG|YBB show how the trains may have alternated turns, starting with the train at station 2. The path corresponding to this record is highlighted in the picture, with trains meeting at station Z.



Connections to Computer Science

If two independent *processes* are running on a computer (say a word processor and an internet browser), since only one *process* can run at any one time, it is necessary to manage which of the *processes* can use the processor at any given point in time. In this question, only one train can move at any point in time, and the other train must stop.

In modern processors, we have multiple-cores, which allow more than one *process* to execute at the same time. Clever algorithms can take advantage of this to speed up the amount of time needed to perform certain tasks. Also, CPUs use *pipelining* to improve efficiency and allow *processes* to use different parts of the CPU in *parallel*.

Country

Czech Republic