

WATERLOO | MATHEMATICS

The CENTRE for EDUCATION in MATHEMATICS and COMPUTING



2014
*Beaver
Computing
Challenge*

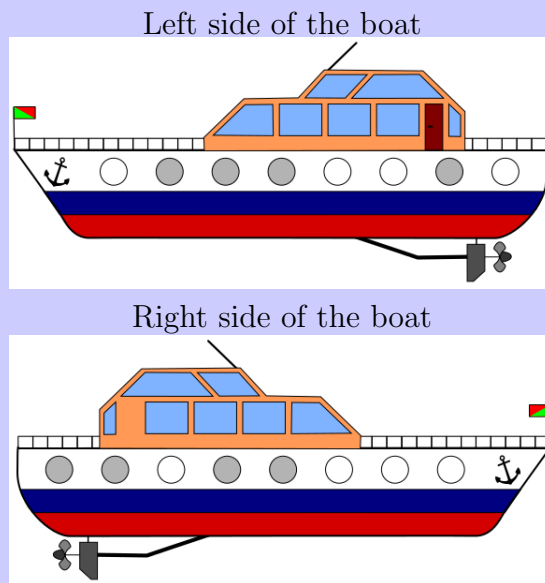
Questions

Part A

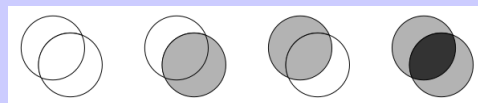
Funny Windows

Story

Glass used to make each window on a boat is either clear or lightly tinted. The left side of a boat has eight circular windows. Directly across from these windows are another eight windows on the right side of the boat. The boat is shown below.



When two pieces of glass overlap, one sees either clear, lightly tinted or darkly tinted glass as shown below.



Question

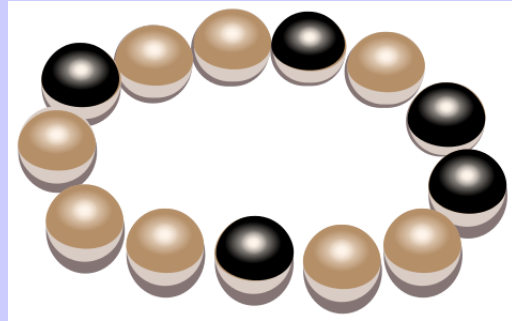
What colours does one see when standing on land looking straight through corresponding windows of the boat above?

(A)	⚓	●	●	○	●	●	○	●	●	
(B)	⚓	○	●	●	●	●	○	●	●	
(C)	⚓	○	●	●	●	●	○	●	●	
(D)	⚓	○	●	●	●	●	○	●	○	

Pearl Bracelet

Story

For the grand ball, a princess wore the bracelet with dark and light pearls shown to the right. After the ball, she unfastened the bracelet between two pearls and put it in a chest. The next evening, she wanted to wear the same bracelet but there were many similar bracelets in the chest.



Question

Which of the following bracelets did the princess wear to the grand ball?

(A)	
(B)	
(C)	
(D)	

Broken Clock

Story

A digital clock displays four digits. Each digit is displayed using seven segments that are each either on or off as shown below.



The clock breaks. Exactly one of the seven segments of one digit does not turn on.

Question



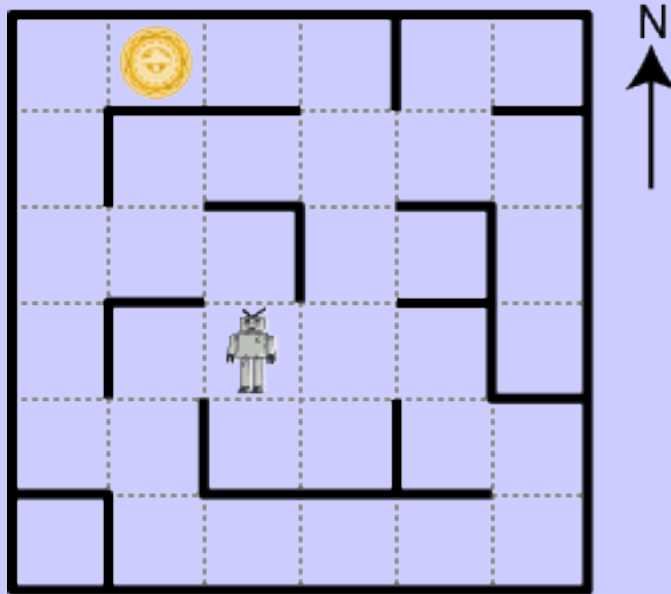
If the broken clock displays the time above, which of the following might be the real time?

- | | |
|-----|--|
| (A) | |
| (B) | |
| (C) | |
| (D) | |

Alien Language

Story

Space beavers are on a mission to find the artifact hidden in the 6-by-6 maze below. Their robot discovered four sequences of alien words. Now, they must program their robot to follow one of these sequences and end on the same space as the artifact. They know the aliens use a different word for each of “move one space North”, “move one space South”, “move one space East” and “move one space West”.



Question

If the robot and artifact are located as shown above, one of the four sequences below is correct. Which is it?

- (A) Ha', Ha', poS, Ha'
- (B) Ha', poS, poS, Ha', nIH, Ha'
- (C) Ha', poS, poS, Ha', Ha', nIH
- (D) Ha', poS, nIH, vI'ogh, Ha', poS

Truth

Story

Beaver Bob only tells the truth on Monday, Wednesday and Friday and always lies on all other days of the week. Today he says: "Tomorrow I will tell the truth."

Question

What day is it?

- (A) Tuesday
- (B) Friday
- (C) Saturday
- (D) Sunday

Part B


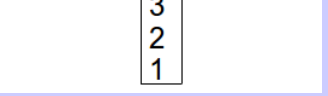
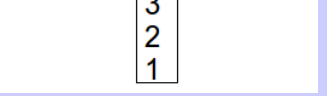
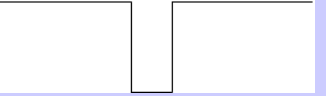
Rabbit Hole

Story

Beavers are going for a stroll in the woods. They walk in a line, one beaver after another.

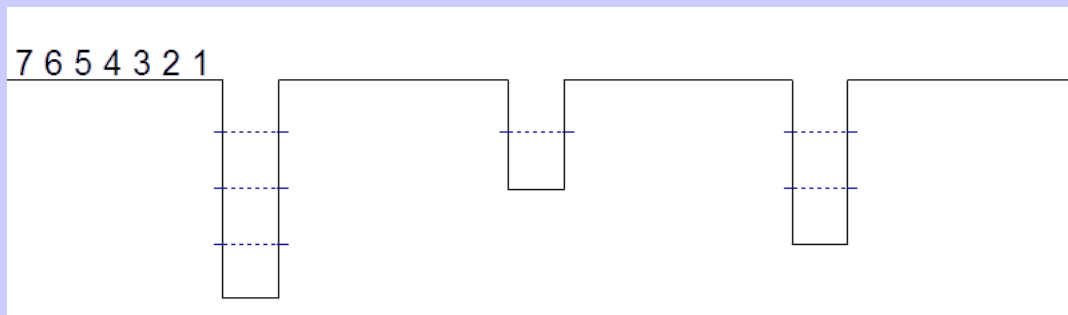
Nasty rabbits have dug holes along the beavers' route.

The holes are deep enough so that some number of beavers will fall in. Once the hole is full of beavers, all the beavers behind the hole walk on top of the beavers in the hole. Then the beavers climb out of the hole from the top to the bottom. The example below involves beavers 1, 2, 3, 4, 5; (1 being the first one in line, and 5 being the last in line) and one hole deep enough for three beavers.

Initially	First three beavers all in the hole	Walking on top of beavers in the hole	Every beaver out of the hole and back in line
<div style="display: flex; justify-content: space-between; width: 100%;"> 5 4 3 2 1 </div> 	<div style="display: flex; justify-content: space-between; width: 100%;"> 5 4 </div> 	<div style="display: flex; justify-content: space-between; width: 100%;"> 5 4 </div> 	<div style="display: flex; justify-content: space-between; width: 100%;"> 1 2 3 5 4 </div> 

Question

If there are 7 beavers (with 1 being the first one in line, and 7 being the last in line), and the first hole encountered holds four beavers, the second hole encountered holds two beavers and the last hole encountered hold three beavers, what is the order of the beavers after all beavers have passed over these three holes?

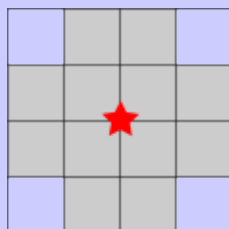


- (A) 3 2 1 6 5 7 4
- (B) 7 4 3 5 6 1 2
- (C) 1 2 3 4 7 5 6
- (D) 2 3 4 1 6 7 5

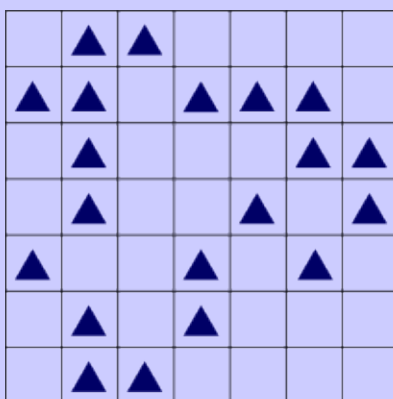
Loudspeakers in a Village

Story

Loudspeakers are set up to announce information to villagers. Each speaker must be located at a point where two grid lines cross. As illustrated below, sound from each speaker reaches twelve grey squares.



The figure below is a map of the village. Triangles represent the locations of houses. It must be possible to hear information from at least one speaker from each house.



Question

What is the fewest number of speakers needed?

- (A) 2
- (B) 3
- (C) 4
- (D) 5

Phone Bills

Story

A communications company stores billing information. There are exactly three charges for each customer (for data, voice and text). Each customer has his or her own unique and distinct phone number. There are two storage options:

OPTION A

Store all the information in one table. Each row in the table corresponds to a data charge, voice charge or text charge.

NAME	PHONE NUMBER	CHARGE
Aki	458-6578	10.00
Aki	458-6578	15.00
Aki	458-6578	10.00
Vlad	235-8998	40.00
Vlad	235-8998	40.00
Vlad	235-8998	30.00
Mia	515-6632	25.00
Mia	515-6632	20.00
Mia	515-6632	20.00

OPTION B

Store the phone number for each customer in one table. Store the charges in a second table in which each row corresponds to a data charge, voice charge or text charge.

NAME	PHONE NUMBER	PHONE NUMBER	CHARGE
Aki	458-6578	458-6578	10.00
Vlad	235-8998	458-6578	15.00
Mia	515-6632	458-6578	10.00
		235-8998	40.00
		235-8998	40.00
		235-8998	30.00
		515-6632	25.00
		515-6632	20.00
		515-6632	20.00

The amount of storage is measured in bytes. Each name requires 128 bytes. Each phone number requires 4 bytes. Each charge requires 4 bytes. These measurements do not depend on how long names are or how big charges are.

Question

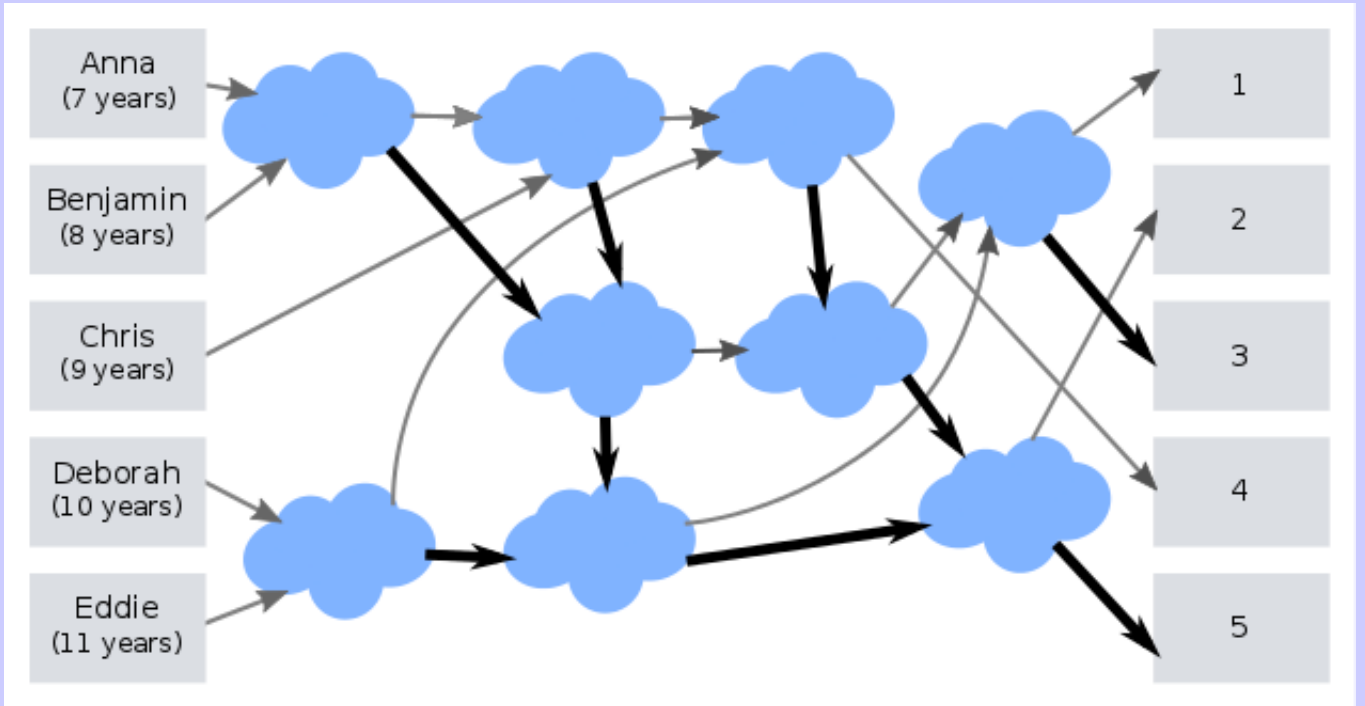
Suppose A and B are the amounts of storage in bytes required by options **A** and **B** respectively. If the company has 1000 cell phone customers, which of the following statements is true?

- (A) $A = B$
- (B) $A < B$
- (C) $A > B$ and $A < 2B$
- (D) $A \geq 2B$

Network Game

Story

Five beavers, Anna (age 7), Benjamin (age 8), Chris (age 9), Deborah (age 10) and Eddie (age 11) are playing a game where they walk through the clouds following arrows shown below. At every cloud, they wait for another beaver to arrive. Then the older beaver leaves along the thick arrow, while the younger one leaves along the thin arrow.



Question

The exits are numbered 1, 2, 3, 4 and 5. Which exit does each beaver reach at the end of the game?

(A)	(B)	(C)	(D)
1: Anna	1: Eddie	1: Benjamin	1: Benjamin
2: Benjamin	2: Deborah	2: Deborah	2: Chris
3: Chris	3: Chris	3: Chris	3: Deborah
4: Deborah	4: Benjamin	4: Anna	4: Anna
5: Eddie	5: Anna	5: Eddie	5: Eddie

Lazy Beaver

Story

A lazy beaver hires five strong beavers. Everyday, each working beaver receives orders to either collect logs from the forest and bring them to the warehouse, or remove logs from the warehouse for processing. Initially, the warehouse has 100 logs. The number of logs added or removed for each beaver is given in the table below.

	Collect	Process
Beaver A	Add 81 logs to warehouse	Remove 81 logs from warehouse
Beaver B	Add 27 logs to warehouse	Remove 27 logs from warehouse
Beaver C	Add 9 logs to warehouse	Remove 9 logs from warehouse
Beaver D	Add 3 logs to warehouse	Remove 3 logs from warehouse
Beaver E	Add 1 log to warehouse	Remove 1 log from warehouse

If a beaver is on vacation, it does not add or remove logs from the warehouse. For example, if Beavers A and D are on vacation, Beaver B is ordered to “Collect” and Beavers C and E are ordered to “Process”, then at the end of the day, the warehouse will have $100 + 27 - 9 - 1 = 117$ logs.

Question

Which of the following orders will leave 168 logs at the end of a day?

- (A) Beavers A, D, and E “Collect”; Beavers B and C “Process”.
- (B) Beavers A and E “Collect”; Beavers B and D “Process”; Beaver C is on vacation.
- (C) Beavers A and B “Collect”; Beavers D, E “Process”; Beaver C is on vacation.
- (D) Beaver A “Collect”; Beavers C, D and E “Process”; Beaver B is on vacation.

Part C

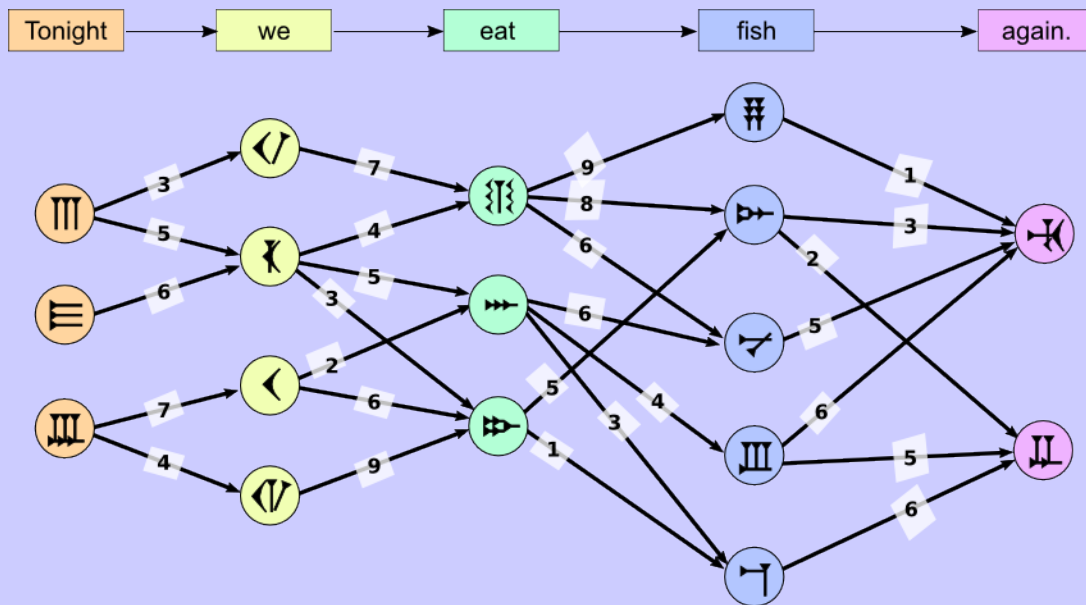
Translation Machine

Story

Betty programs a machine that translates an English sentence to a Beavarian sentence one word at a time. However, there are several possible Beavarian words for each English word!

Betty noticed that different words occur next to each other at different rates. For example, “smart beaver” is more common than “intelligent beaver.” She gives scores for word pairs: the higher the score, the more common the word pair is.

An English sentence with five words must be translated into five Beavarian symbols. In the picture below, arrows labelled with scores connect all valid word pairs. The total score for a translation is the sum of the scores of the four arrows used.



Question

What is the highest possible total score for a translation of this sentence?

- (A) 18
- (B) 21
- (C) 22
- (D) 23

Strange Words

Story

Beavers consider words containing only the letters a , b , c . There are three different operations we can apply to such words:

- Operation 1: Replace every a with the sequence aa .
- Operation 2: Replace some b with c .
- Operation 3: Insert the letter c anywhere in the word.

We can use any of these operations in any order, and may use an operation many times. For example, if we have the word abc then using Operation 2 we could get acb , then using Operation 1 we could get $aacb$, and finally using Operation 1 we could get $aaaacb$.

Question

Which of the following words is impossible to get if we start with $aabbbbaabbccbbabbc$?

- (A) $aaaabbcbaaaabbcccbbaabbc$
- (B) $accabcbcaabbccbbaccc$
- (C) $aaaabccbbaaaabbccbaabcc$
- (D) $accccaaaccccaacaabbccbbaabbc$

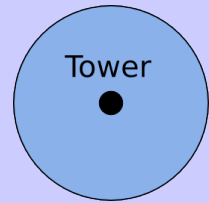
Robust Network

Story

The Beaver TeleCompany wants to place cellphone towers on Windy Island.

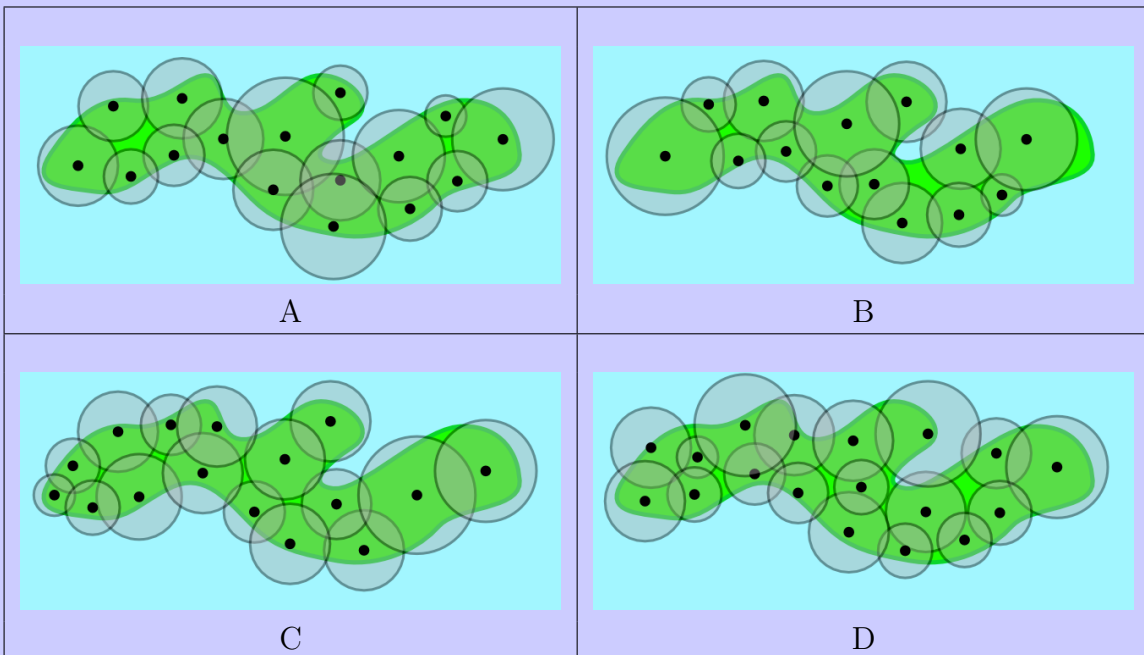
A cellphone tower's coverage area is a circle centered around it. Two towers are connected if their coverage areas overlap. Furthermore, two towers can communicate through a sequence of towers where consecutive towers are connected.

The wind on the island often breaks towers. With any single tower broken, it must be possible for any two of the remaining towers to communicate.



Question

Of the choices below, how should the towers be placed?



Height Game

Story

Young beavers Amy, Beavy, Cuttree, Diggy, and Eary, are all different heights. They line up, one after another, facing the same way, in some order. Then each beaver finds all the other beavers that are taller than himself/herself. He/she counts how many of these taller beavers are in front of him/her and how many are behind him/her. The results are shown in the following table:

Name	Number of taller beavers	
	in front	behind
Amy	1	2
Beavy	3	1
Cuttree	1	0
Diggy	0	0
Eary	2	0

Question

In what order are they standing in, from the front of the line to the back of the line?

- (A) Diggy, Cuttree, Amy, Beavy, Eary
- (B) Diggy, Amy, Cuttree, Beavy, Eary
- (C) Amy, Cuttree, Diggy, Eary, Beavy
- (D) Diggy, Amy, Eary, Beavy, Cuttree

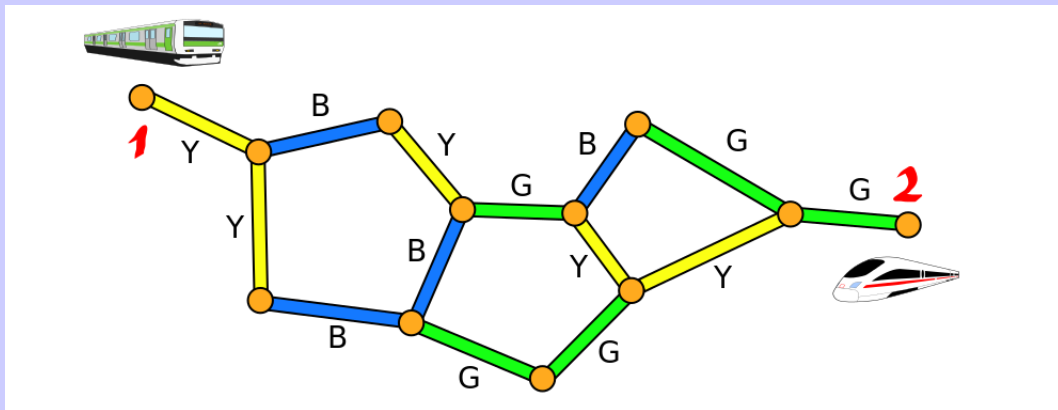
Express Trains

Story

Two trains are going towards each other starting from stations 1 and 2. The map below shows all the stations and coloured rail tracks between them.

At every moment, one of the trains is moving, and the other train is stopped at some station. While a train moves, the colour of the rail track (either Blue, Green or Yellow, marked as B, G or Y, respectively) it is using is recorded. Unfortunately, the record does not store which of the trains was moving.

For example, the record BG can either mean that one of the trains passed over a Blue and then a Green rail track, or it could mean that one train passed over a Blue rail track, and then the other train passed over a Green rail track.



Question

After some number of moves, the two trains meet. One of the following records the trains' movements up to the moment of meeting. Which is it?

- (A) GYGBGYBB
- (B) YYBYGGBG
- (C) GBYBYGY
- (D) YBBYBYY