Gauss Contest

Grade 8

(The Grade 7 Contest is on the reverse side)

Wednesday, May 15, 2013
(in North America and South America)

Thursday, May 16, 2013
(outside of North America and South America)

UNIVERSITY OF **WATERLOO**   **WATERLOO MATHEMATICS**   **Deloitte.**

Time: 1 hour   ©2012 University of Waterloo

Calculators are permitted.

Instructions

1. Do not open the contest booklet until you are told to do so.

2. You may use rulers, compasses and paper for rough work.

3. Be sure that you understand the coding system for your answer sheet. If you are not sure, ask your teacher to explain it.

4. This is a multiple-choice test. Each question is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. When you have made your choice, enter the appropriate letter for that question on your answer sheet.

5. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
   There is *no penalty* for an incorrect answer.
   Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

6. Diagrams are *not* drawn to scale. They are intended as aids only.

7. When your supervisor instructs you to start, you will have *sixty* minutes of working time.

The name, school and location of some top-scoring students will be published on the Web site, http://www.cemc.uwaterloo.ca. You will also be able to find copies of past Contests and excellent resources for enrichment, problem solving and contest preparation.

Grade 8

Part A: Each correct answer is worth 5.

1. The value of $10^2 + 10 + 1$ is

   (A) 101   **(B)** 1035   **(C)** 1011   **(D)** 111   **(E)** 31

2. The value of $15 - 3 - 15$ is

   (A) -18 **(B)** -15 **(C)** 3   **(D)** -3 **(E)** -33

3. The smallest number in the set $\left\{ \frac{1}{2}, \frac{2}{3}, \frac{1}{4}, \frac{5}{6}, \frac{7}{12} \right\}$ is

   (A) $\frac{1}{2}$ **(B)** $\frac{2}{3}$ **(C)** $\frac{1}{4}$ **(D)** $\frac{5}{6}$ **(E)** $\frac{7}{12}$

4. Ahmed is going to the store. One quarter of the way to the store, he stops to talk with Kee. He then continues for 12 km and reaches the store. How many kilometres does he travel altogether?

   (A) 15   **(B)** 16   **(C)** 24

   (D) 48   **(E)** 20
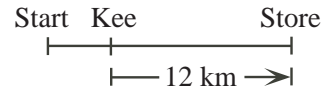
5. Jarek multiplies a number by 3 and gets an answer of 90. If instead, he divides the number by 3, what answer does he get?

   (A) 5   **(B)** 10   **(C)** 30   **(D)** 60   **(E)** 270

6. What number goes in the box so that $10 \times 20 \times 30 \times 40 \times 50 = 100 \times 2 \times 300 \times 4 \times \square$?

   (A) 0.5   **(B)** 5   **(C)** 50   **(D)** 500   **(E)** 5000

7. Each letter of the English alphabet is written on a separate tile and placed in a bag. Alonso draws one letter at random from the bag. What is the probability that Alonso draws a letter that is in his name?

   (A) $\frac{1}{26}$ **(B)** $\frac{4}{26}$ **(C)** $\frac{5}{26}$ **(D)** $\frac{2}{26}$ **(E)** $\frac{3}{26}$

8. Mathy Manuel's autograph was once worth $100. The autograph then dropped 30% in value. If it then increased by 40%, its value would be

   (A) $98   **(B)** $48   **(C)** $100   **(D)** $78   **(E)** $90

9. The point $(-2, -3)$ is reflected in the x-axis. What are the coordinates of its image after the reflection?

   (A) $(2, -3)$ **(B)** $(3, -2)$ **(C)** $(2, 3)$

   (D) $(-3, -2)$ **(E)** $(-2, 3)$
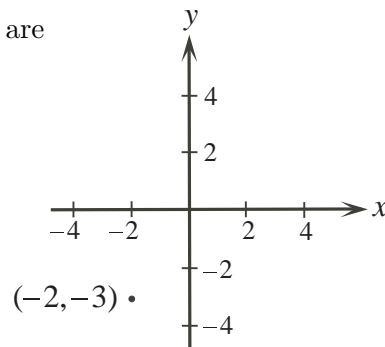
10. The ratio of the value of four nickels (5¢ coins) to the value of six dimes (10¢ coins) to the value of two quarters (25¢ coins) can be written as

    (A) $4:6:2$ **(B)** $2:6:5$ **(C)** $2:3:1$ **(D)** $6:4:2$ **(E)** $1:2:3$

Part B: Each correct answer is worth 6.

11. If $x = 4$ and $3x + 2y = 30$, what is the value of y?

    (A) 18   **(B)** 6   **(C)** 3   **(D)** 4   **(E)** 9

12. The value of $(2^3)^2 - 4^3$ is

    (A) 0   **(B)** -8 **(C)** 4   **(D)** 10   **(E)** 12

13. The Summer Olympics are held once every 4 years. During an 18 year period, what is the largest number of Summer Olympics that could be held?

    (A) 3   **(B)** 4   **(C)** 5   **(D)** 6   **(E)** 7

14. A cube has a surface area of $54\,\text{cm}^2$. The volume of the cube, in cm^3, is

    (A) 81   **(B)** 343   **(C)** 18   **(D)** 27   **(E)** 729

15. When $10\,000$ is divided by 13, the remainder is 3. Which one of the following numbers also gives a remainder of 3 when divided by 13?

    (A) 9997   **(B)** $10\,003$ **(C)** $10\,013$ **(D)** $10\,010$ **(E)** $10\,016$

16. A family has 3 children. It is equally likely that a child is a boy as it is that a child is a girl. What is the probability that the 3 children are all girls?

    (A) $\frac{2}{3}$ **(B)** $\frac{1}{4}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{1}{3}$ **(E)** $\frac{1}{8}$

17. $PQRS$ is a rectangle with diagonals PR and QS, as shown. The value of y is

    (A) 30   **(B)** 40   **(C)** 45

    (D) 50   **(E)** 60
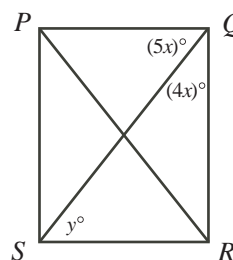
18. Sally is asked to multiply $\frac{2}{3}$ and $1\frac{1}{2}$. Jane is asked to add them. The difference between Sally's answer and Jane's answer is

    (A) $\frac{4}{15}$ **(B)** $1\frac{1}{6}$ **(C)** 0   **(D)** $1\frac{3}{5}$ **(E)** $\frac{5}{6}$

19. Serena colours the hexagons on the tiling shown. If two hexagons share a side, she colours them with different colours. What is the least number of colours that she can use to colour all of the hexagons?

    (A) 4   **(B)** 6   **(C)** 7

    (D) 2   **(E)** 3
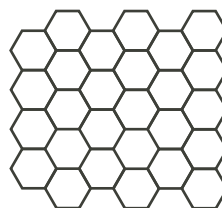
20. Christina and Frieda want to buy the same book. Christina has $\frac{3}{4}$ of the money needed to buy the book and Frieda has half of the money needed to buy the book. If the book was $3 cheaper, then together they would have exactly enough money to buy 2 copies of the book. What is the original price of the book?

    (A) $4 **(B)** $16 **(C)** $12 **(D)** $10 **(E)** $8

Part C: Each correct answer is worth 8.

21. An *arithmetic sequence* is a sequence in which each term after the first is obtained by adding a constant to the previous term. For example, $2, 4, 6, 8$ and $1, 4, 7, 10$ are arithmetic sequences.

    In the grid shown, the numbers in each row must form an arithmetic sequence and the numbers in each column must form an arithmetic sequence. The value of x is

5			
			1211
		1013	
23	x		

    (A) 17 **(B)** 619 **(C)** 515

    (D) 446 **(E)** 793

22. In right-angled, isosceles triangle FGH, $FH = \sqrt{8}$. Arc FH is part of the circumference of a circle with centre G and radius GH, as shown. The area of the shaded region is

    (A) $\pi - 2$ **(B)** $4\pi - 2$ **(C)** $4\pi - \frac{1}{2}\sqrt{8}$

    (D) $4\pi - 4$ **(E)** $\pi - \sqrt{8}$
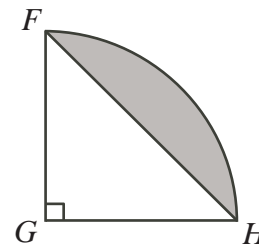
23. Greg, Charlize, and Azarah run at different but constant speeds. Each pair ran a race on a track that measured 100 m from start to finish. In the first race, when Azarah crossed the finish line, Charlize was 20 m behind. In the second race, when Charlize crossed the finish line, Greg was 10 m behind. In the third race, when Azarah crossed the finish line, how many metres was Greg behind?

    (A) 20 **(B)** 25 **(C)** 28 **(D)** 32 **(E)** 40

24. In any triangle, the length of the longest side is less than half of the perimeter. All triangles with perimeter 57 and integer side lengths x, y, z, such that $x < y < z$ are constructed. How many such triangles are there?

    (A) 68 **(B)** 61 **(C)** 75 **(D)** 56 **(E)** 27

25. At the beginning of the winter, there were at least 66 students registered in a ski class. After the class started, eleven boys transferred into this class and thirteen girls transferred out. The ratio of boys to girls in the class was then $1 : 1$. Which of the following is not a possible ratio of boys to girls before the transfers?

    (A) $4 : 7$ **(B)** $1 : 2$ **(C)** $9 : 13$ **(D)** $5 : 11$ **(E)** $3 : 5$