The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING
cemc.uwaterloo.ca

Fryer Contest

(Grade 9)

Thursday, April 18, 2013
(in North America and South America)

Friday, April 19, 2013
(outside of North America and South America)

Do not open this booklet until instructed to do so.

Time: 75 minutes
Calculators are permitted

Number of questions: 4
Each question is worth 10 marks

Parts of each question can be of two types:

1. **SHORT ANSWER** parts indicated by

   - worth 2 or 3 marks each
   - full marks given for a correct answer which is placed in the box
   - **part marks awarded only if relevant work** is shown in the space provided

2. **FULL SOLUTION** parts indicated by

   - worth the remainder of the 10 marks for the question
   - **must be written in the appropriate location** in the answer booklet
   - marks awarded for completeness, clarity, and style of presentation
   - a correct solution poorly presented will not earn full marks

WRITE ALL ANSWERS IN THE ANSWER BOOKLET PROVIDED.

- Extra paper for your finished solutions supplied by your supervising teacher must be inserted into your answer booklet. Write your name, school name, and question number on any inserted pages.
- Express calculations and answers as exact numbers such as $\pi + 1$ and $\sqrt{2}$, etc., rather than as $4.14\ldots$ or $1.41\ldots$, except where otherwise indicated.

Do not discuss the problems or solutions from this contest online for the next 48 hours.

The name, grade, school and location of some top-scoring students will be published on our Web site, http://www.cemc.uwaterloo.ca. In addition, the name, grade, school and location, and score of some top-scoring students may be shared with other mathematical organizations for other recognition opportunities.

TIPS:

1. Please read the instructions on the front cover of this booklet.
2. Write all answers in the answer booklet provided.
3. For questions marked  , place your answer in the appropriate box in the answer booklet and **show your work**.
4. For questions marked  , provide a well-organized solution in the answer booklet. Use mathematical statements and words to explain all of the steps of your solution. Work out some details in rough on a separate piece of paper before writing your finished solution.
5. Diagrams are *not* drawn to scale. They are intended as aids only.

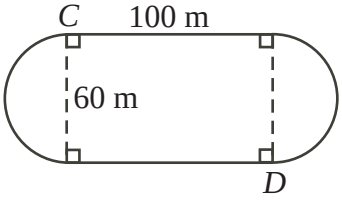
1. Ann, Bill and Cathy went bowling. In bowling, each score is a whole number.

(a) In Ann's first game, her score was 103. In her second game, her score was 117. What was her average score for these two games?

(b) In his first two games, Bill's scores were 108 and 125. His average score after three games was 115. What was his score in the third game?

(c) After three games, Cathy's average score was 113. She scored the same in her fifth game as she did in her fourth game. Is it possible for her average score on these five games to be 120? Explain why or why not.

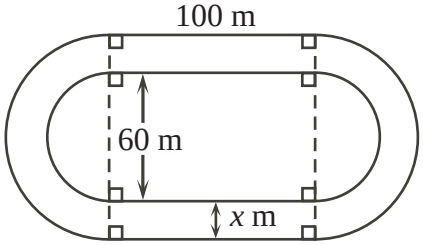
2. The outside of a field consists of two straight sides each of length 100 m joined by two semi-circular arcs each of diameter 60 m, as shown below.



(a) Determine the perimeter of the field.

(b) Amy and Billais run from point C to point D. Amy runs along the perimeter of the field, and Billais runs in a straight line from C to D. Rounded to the nearest metre, how much farther does Amy travel than Billais?

(c) The diagram below shows a track of constant width x m built around the field. The outside of the track has two straight sides each of length 100 m joined by two semi-circular arcs. The perimeter of the outside of the track is 450 m. Determine the value of x rounded to the nearest whole number.



3. The *sum of the digits* of 2013 is $2 + 0 + 1 + 3 = 6$. If the sum of the digits of a positive integer is divisible by 3, then the number is divisible by 3. Also, if a positive integer is divisible by 3, then the sum of its digits is divisible by 3.

(a) List all values for the digit A such that the four-digit number $51A3$ is divisible by 3.

(b) List all values for the digit B such that the four-digit number $742B$ is divisible by both 2 and 3 (that is, is divisible by 6).

(c) Find all possible pairs of digits P and Q such that the number $1234PQPQ$ is divisible by 15.

(d) Determine the number of pairs of digits C and D for which the product $2CC \times 3D5$ is divisible by 12.

4. A dot starts on the xy-plane at $(0, 0)$ and makes a series of moves.
In each move, the dot travels one unit either left ($\leftarrow$), right ($\rightarrow$), up ($\uparrow$), or down ($\downarrow$).

Five of the many different ways that the dot could end at the point $(1, 1)$ are $\uparrow\rightarrow$, $\rightarrow\uparrow$, $\uparrow\downarrow\rightarrow\uparrow$, $\uparrow\uparrow\rightarrow\downarrow$, and $\uparrow\rightarrow\rightarrow\leftarrow$.

(a) In how many different ways can the dot end at the point $(1, 0)$ in 4 or fewer moves?

(b) At how many different points can the dot end in exactly 4 moves?

(c) Determine, with justification, the number of integers k with $k \leq 100$ for which the dot can end at the point $(-7, 12)$ in exactly k moves.

(d) The dot can end at exactly 2304 points in exactly 47 moves. Determine, with justification, the number of points at which the dot can end in exactly 49 moves.

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Thank you for writing the 2013 Fryer Contest!
In 2012, more than 13 000 students from around the world registered to write the Fryer, Galois and Hypatia Contests.

Encourage your teacher to register you for the Canadian Intermediate Mathematics Contest or the Canadian Senior Mathematics Contest, which will be written in November 2013.

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