The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING
cemc.uwaterloo.ca

Fermat Contest

(Grade 11)

Thursday, February 21, 2013
(in North America and South America)

Friday, February 22, 2013
(outside of North America and South America)

UNIVERSITY OF WATERLOO WATERLOO MATHEMATICS Deloitte.

Time: 60 minutes
Calculators are permitted
Instructions

1. Do not open the Contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name and city/town in the box in the upper right corner.
5. **Be certain that you code your name, age, sex, grade, and the Contest you are writing in the response form. Only those who do so can be counted as eligible students.**
6. This is a multiple-choice test. Each question is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. After making your choice, fill in the appropriate circle on the response form.
7. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
   There is *no penalty* for an incorrect answer.
   Each unanswered question is worth 2, to a maximum of 10 unanswered questions.
8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor tells you to begin, you will have *sixty* minutes of working time.

Do not discuss the problems or solutions from this contest online for the next 48 hours.

The name, grade, school and location, and score range of some top-scoring students will be published on our website, http://www.cemc.uwaterloo.ca. In addition, the name, grade, school and location, and score of some top-scoring students may be shared with other mathematical organizations for other recognition opportunities.

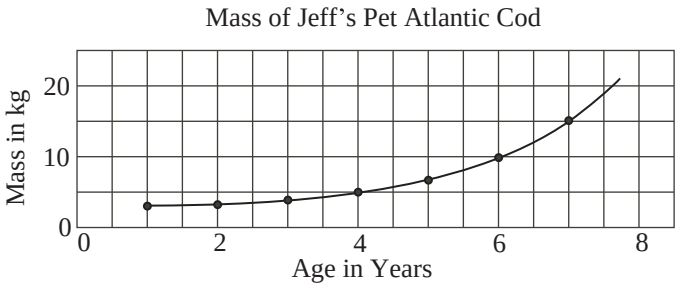
Scoring:	There is *no penalty* for an incorrect answer. Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

Part A: Each correct answer is worth 5.

1. What is the value of $\dfrac{10^2 + 6^2}{2}$?

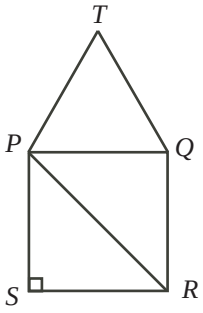
   (A) 16 **(B)** 86 **(C)** 34 **(D)** 68 **(E)** 128

2. The graph shows the mass, in kilograms, of Jeff's pet Atlantic cod, given its age in years. What is the age of the cod when its mass is 15 kg?



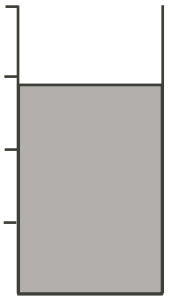
   (A) 3 **(B)** 7 **(C)** 4 **(D)** 6 **(E)** 5

3. In the diagram, $PQRS$ is a square and PTQ is an equilateral triangle. What is the measure of $\angle TPR$?



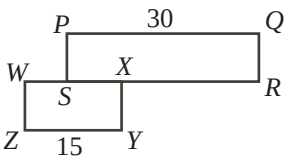
   (A) 90° **(B)** 105° **(C)** 120°
   (D) 150° **(E)** 75°

4. A large cylinder can hold 50 L of chocolate milk when full. The tick marks show the division of the cylinder into four parts of equal volume. Which of the following is the best estimate for the volume of chocolate milk in the cylinder as shown?



   (A) 24 L **(B)** 28 L **(C)** 30 L
   (D) 36 L **(E)** 40 L

5. In the diagram, rectangle $PQRS$ has $PQ = 30$ and rectangle $WXYZ$ has $ZY = 15$. If S is on WX and X is on SR so that $SX = 10$, then WR equals



   (A) 20 **(B)** 25 **(C)** 55
   (D) 45 **(E)** 35

6. If $x = 11$, $y = 8$, and $2x + 3z = 5y$, what is the value of z?

(A) 6 **(B)** $\frac{62}{3}$ **(C)** 13 **(D)** 15 **(E)** $\frac{46}{5}$

7. If $(x + a)(x + 8) = x^2 + bx + 24$ for all values of x, then $a + b$ equals

(A) 32 **(B)** 144 **(C)** 40 **(D)** 14 **(E)** 16

8. Which number from the set $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$ must be removed so that the mean (average) of the numbers remaining in the set is 6.1?

(A) 4 **(B)** 5 **(C)** 6 **(D)** 7 **(E)** 8

9. The regular price for a bicycle is \$320. The bicycle is on sale for 20% off. The regular price for a helmet is \$80. The helmet is on sale for 10% off. If Sandra bought both items on sale, what is her percentage savings on the total purchase?

(A) 18% **(B)** 12% **(C)** 15% **(D)** 19% **(E)** 22.5%

10. $PQRS$ is a square. The midpoint of PQ is M and the midpoint of RS is N. If the perimeter of rectangle $PMNS$ is 36, the area of square $PQRS$ is

(A) 81 **(B)** 72 **(C)** 324 **(D)** 144 **(E)** 36

Part B: Each correct answer is worth 6.

11. On Monday, Ramya read $\frac{1}{5}$ of a 300 page novel. On Tuesday, she read $\frac{4}{15}$ of the remaining pages. How many pages did she read in total on Monday and Tuesday?

(A) 124 **(B)** 60 **(C)** 252 **(D)** 80 **(E)** 64

12. An integer m is chosen at random from the list $-9, -7, -5, -3, -1, 1, 3, 5, 7, 9$. The probability that $m^4 > 100$ is

(A) $\frac{1}{5}$ **(B)** $\frac{3}{10}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{2}{5}$ **(E)** $\frac{3}{5}$

13. If $512^x = 64^{240}$, then x equals

(A) 80 **(B)** 30 **(C)** 360 **(D)** 160 **(E)** 237

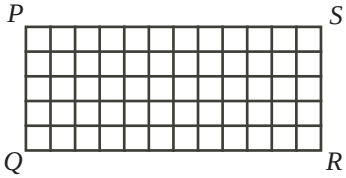
14. In a school fundraising campaign, 25% of the money donated came from parents. The rest of the money was donated by teachers and students. The ratio of the amount of money donated by teachers to the amount donated by students was 2 : 3. The ratio of the amount of money donated by parents to the amount donated by students was

(A) 20 : 9 **(B)** 5 : 6 **(C)** 5 : 9 **(D)** 1 : 2 **(E)** 5 : 12

15. The cookies in a cookie jar contain a total of 100 raisins. All but one of the cookies are the same size and contain the same number of raisins. One cookie is larger and contains one more raisin than each of the others. The number of cookies in the jar is between 5 and 10, inclusive. How many raisins are in the larger cookie?

(A) 10 **(B)** 11 **(C)** 20 **(D)** 17 **(E)** 12

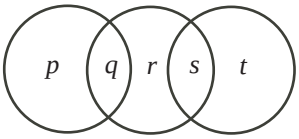
16. Rectangle $PQRS$ is divided into 60 identical squares, as shown. The length of the diagonal of each of these squares is 2. The length of QS is closest to



(A) 18 **(B)** 13 **(C)** 26

(D) 24 **(E)** 17

17. In the diagram, p, q, r, s, and t represent five consecutive integers, not necessarily in order. The two integers in the leftmost circle add to 63. The two integers in the rightmost circle add to 57. What is the value of r?



(A) 24 **(B)** 28 **(C)** 20

(D) 42 **(E)** 30

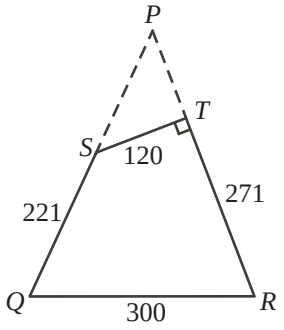
18. If m, n and p are positive integers with $m + \dfrac{1}{n + \dfrac{1}{p}} = \dfrac{17}{3}$, the value of n is

(A) 3 **(B)** 4 **(C)** 1 **(D)** 17 **(E)** 13

19. There are two ways of choosing six different numbers from the list $1, 2, 3, 4, 5, 6, 7, 8, 9$ so that the product of the six numbers is a perfect square. Suppose that these two perfect squares are m^2 and n^2, with m and n positive integers and $m \neq n$. What is the value of $m + n$?

(A) 108 **(B)** 11 **(C)** 61 **(D)** 56 **(E)** 144

20. Isosceles triangle PQR has $PQ = PR$ and $QR = 300$. Point S is on PQ and T is on PR so that ST is perpendicular to PR, $ST = 120$, $TR = 271$, and $QS = 221$. The area of quadrilateral $STRQ$ is

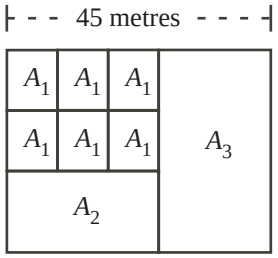


(A) 21 275 **(B)** 40 605 **(C)** 46 860

(D) 54 000 **(E)** 54 603

Part C: Each correct answer is worth 8.

21. A farmer has a rectangular field with width 45 metres. He divides the field into smaller rectangular animal enclosures in three different sizes, as shown.



Each enclosure labelled A_1 has the same dimensions. Also, the area of the enclosure labelled A_2 is 4 times the area of A_1, and the area of the enclosure labelled A_3 is 5 times the area of A_1. The lines in the diagram represent fences. The total length of all the fences is 360 metres. The area of A_1, in square metres, is closest to

(A) 143.4 **(B)** 150.0 **(C)** 175.2 **(D)** 162.7 **(E)** 405.0

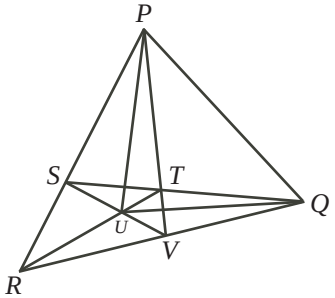
22. Megan and Shana race against each other with the winner of each race receiving x gold coins and the loser receiving y gold coins. (There are no ties and x and y are integers with $x > y > 0$.) After several races, Megan has 42 coins and Shana has 35 coins. Shana has won exactly 2 races. The value of x is

(A) 3 **(B)** 7 **(C)** 5 **(D)** 6 **(E)** 4

23. One bag contains 2 red marbles and 2 blue marbles. A second bag contains 2 red marbles, 2 blue marbles, and g green marbles, with $g > 0$. For each bag, Maria calculates the probability of randomly drawing two marbles of the same colour in two draws from that bag, without replacement. (Drawing two marbles *without replacement* means drawing two marbles, one after the other, without putting the first marble back into the bag.) If these two probabilities are equal, then the value of g is

(A) 4 **(B)** 5 **(C)** 6 **(D)** 7 **(E)** 8

24. In the diagram, $\triangle PQR$ has S on PR and V on RQ. Segments QS and PV intersect at T. Segments RT and SV intersect at U. If the area of $\triangle RST$ is 55, the area of $\triangle RTV$ is 66, and the area of $\triangle RSV$ is 77, then the area of $\triangle PQU$ is



(A) 869 **(B)** 836 **(C)** 840

(D) 864 **(E)** 847

25. For how many odd integers k between 0 and 100 does the equation

$$2^{4m^2} + 2^{m^2-n^2+4} = 2^{k+4} + 2^{3m^2+n^2+k}$$

have exactly two pairs of positive integers (m, n) that are solutions?

(A) 17 **(B)** 20 **(C)** 19 **(D)** 18 **(E)** 21

The CENTRE for EDUCATION in MATHEMATICS and COMPUTING

cemc.uwaterloo.ca

For students...

Thank you for writing the 2013 Fermat Contest!
In 2012, more than 75 000 students around the world registered to write the Pascal, Cayley and Fermat Contests.

Encourage your teacher to register you for the Hypatia Contest which will be written in April.

Visit our website to find

- More information about the Hypatia Contest
- Free copies of past contests
- Workshops to help you prepare for future contests
- Information about our publications for mathematics enrichment and contest preparation

For teachers...

Visit our website to

- Register your students for the Fryer, Galois and Hypatia Contests which will be written in April
- Learn about our face-to-face workshops and our resources
- Find your school contest results