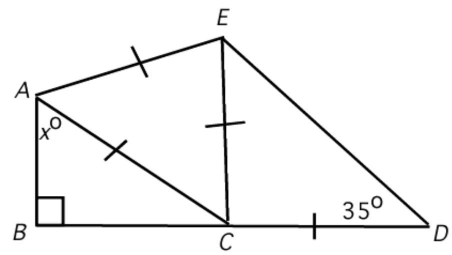
The CENTRE for EDUCATION in MATHEMATICS and COMPUTING
presents the
2013 Canadian Team Mathematics Contest (CTMC)

Team Problems

1. In the diagram, AB is perpendicular to BD. Also, $\angle CDE = 35°$ and $\triangle ACE$ is equilateral. Determine the value of x.



2. At Laurel Creek, a season's pass costs \$45 per adult and \$30 per child. A season's family pass costs \$110 per family. For a family of one adult and three children, how much does the season's family pass save as compared to buying individual passes?

3. If $3 : 5 = x : y$ and $6 : 11 = x : z$ then find $y : z$.

4. If $(a, b)\nabla(c, d) = ad - bc$, determine the value of $(20, 17)\nabla(19, 15)$.

5. Three brothers, Jamie who is 13, Ian who is 11 and Riley who is 7 have their birthdays on the same day. Their dad notices that their ages are all prime numbers. While Jamie is between 10 and 50 years old, how many times will their ages all be prime numbers?

6. Determine all values of x such that $3^{x^2+2x-15} = 1$.

7. Determine the value of $x > 0$ such that $x\%$ of 3 equals 9% of x^2.

8. A triangle has angles in the ratio $2 : 3 : 4$. The smallest angle is bisected, forming two new triangles. One of those two triangles contains an obtuse angle. Determine the ratio of the angles of this triangle.

9. Three numbers a, b and c satisfy

$$\begin{aligned} 3a + 5b - c &= -2 \\ a - 2b + 3c &= 13 \\ 2a + 3b + 4c &= 13 \end{aligned}$$

Determine the value of $a + b + c$.

10. When the number 1 is divided by 3, the remainder is 1. Likewise, the remainder is 1 when the number 1 is divided by 5 and by 7. What is the next smallest positive integer that has a remainder of 1 when divided by 3, by 5, and 7?

11. A geometric sequence is a sequence in which each term after the first is obtained from the previous term by multiplying by a constant. For example $3, 6, 12, 24, \ldots$ is a geometric sequence. If $\sqrt{5}$ and $\sqrt{10}$ are the first 2 terms of a geometric sequence and the 10th term is $\sqrt{m}$, determine the value of m.

12. An egg is thrown off a platform 100 m in the air. From the time t in seconds when thrown until it hits the ground, the egg's height off the ground in metres follows the formula
$h = -5t^2 + 30t + 100$.
Find the difference between the maximum height and the minimum height of the egg in the first 5 seconds.

13. How many integers x satisfy the inequality $x^4 - 29x^2 + 100 \leq 0$?

14. A vertical line is drawn through a big square, dividing the area in the ratio $2 : 3$. Next a horizontal line is drawn through the big square. Now the big square is comprised of a small square, a medium-sized square, and two rectangles. Determine the ratio of the area of the small square to the area of the medium-sized square.

15. A and B are two distinct points on the Cartesian plane. How many different points C are there so $\triangle ABC$ is either an equilateral triangle or a right isosceles triangle?

16. A class of 23 students had an average of 88% on a test. A second class of 27 students had an average of 82%. When a marking error was discovered, Nerissa (from the first class) and Maria (from the second class) each had their mark changed to 98%. After this, the combined average of all of the students in the two classes was then 85%. What was the average of Nerissa's and Maria's original marks?

17. If x and y are integers such that
$$3^{x+2}12^3 = 4^{x-2}9^y,$$
determine the value of y.

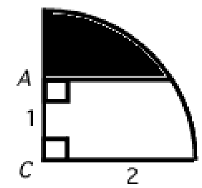
18. Riley flies in his plane from Breslau to Winnipeg, a distance of 1500 km, into a constant headwind of 50 km/h. Immediately upon arrival he realizes he has forgotten his brother Jamie in Breslau so he flies back to Breslau with the 50 km/h wind now behind him. The total trip takes 12.5 hours. How fast does the plane fly without a wind present?

19. A "sideways adder" works when you add the digits both downwards and to the right.
$$\begin{array}{cc||c} 1 & 3 & 4 \\ 7 & 2 & 9 \\ \hline\hline 8 & 5 & + \end{array}$$
Note that $13 + 72 = 85$ and $71 + 23 = 94$.

There is a sideways adder that uses all of the digits from 1 to 9 exactly once. The digit 8 is given. Determine the rest of the digits.
$$\begin{array}{ccc||c} & \square & \square & \square \\ & \square & \square & \square \\ \hline\hline \square & \square & 8 & + \end{array}$$

20. How many different ways can numbers a, b, c be chosen from the set $\{1,2,3,4,\ldots,10\}$ so that $a < b < c$ and $c - b = b - a$?

21. In the diagram, part of a quarter circle of radius 2 is shaded. The length of AC is 1. Determine the area of the shaded region.



22. The contest scores of a number of students are entered from highest to lowest in consecutive rows of a table beginning in row 1. Many students may have the same score, so many rows in the table contain the same value. For the six highest scores, the chart below shows the average row number in the first table for each score. How many students had a score of 111?

Score	*Average Row Number*
148	2
137	4.5
128	7.5
127	11
111	15
104	23

23. The curves $y = \sqrt{2}\sin x$ and $y = \sin 90°$ intersect 9 times when $0° \leq x \leq k°$. Find the smallest possible value for k.

24. In the sequence $1, 2, 3, 1, 4, 3, 7, 4, 11, 7, 18, \ldots, 1364, 3571$, the first two terms are 1 and 2. Every term after the second is formed by combining the previous two terms, alternating between adding and finding the positive difference. Find the sum of this sequence.

25. A game is played with one regular, fair 20-sided die with faces labelled from 1 to 20. To begin the game, Ping throws the die once to determine a "target" number. Then, Ping throws the die again. If Ping rolls the "target" number again, she wins the game. If Ping rolls one more or one less than the target number, then she loses. If Ping rolls anything else, she rolls again. She keeps on rolling until she wins or loses. What is the probability that she wins the game?