The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING

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Canadian Intermediate Mathematics Contest

Thursday, November 21, 2013
(in North America and South America)

Friday, November 22, 2013
(outside of North America and South America)

Time: 2 hours

Calculators are permitted, provided they are non-programmable and without graphic displays.

Do not open this booklet until instructed to do so.
There are two parts to this paper.

PART A

1. This part consists of six questions, each worth 5 marks.
2. **Enter the answer in the appropriate box in the answer booklet.**
For these questions, full marks will be given for a correct answer which is placed in the box. Part marks will be awarded **only if relevant work** is shown in the space provided in the answer booklet.

PART B

1. This part consists of three questions, each worth 10 marks.
2. **Finished solutions must be written in the appropriate location in the answer booklet.** Rough work should be done separately. If you require extra pages for your finished solutions, they will be supplied by your supervising teacher. Insert these pages into your answer booklet. Be sure to write your name, school name and question number on any inserted pages.
3. Marks are awarded for completeness, clarity, and style of presentation. A correct solution, poorly presented, will not earn full marks.

NOTES:

The questions in each part are arranged roughly in order of increasing difficulty.
The early problems in Part B are likely easier than the later problems in Part A.
At the completion of the contest, insert your student information form inside your answer booklet.

Do not discuss the problems or solutions from this contest online for the next 48 hours.

The name, grade, school and location, and score range of some top-scoring students will be published on the Web site, http://www.cemc.uwaterloo.ca. In addition, the name, grade, school and location, and score of some students may be shared with other mathematical organizations for other recognition opportunities.

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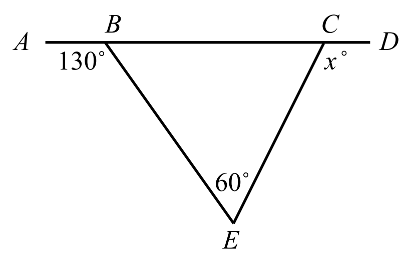
NOTE:	1.	Please read the instructions on the front cover of this booklet.
	2.	Write solutions in the answer booklet provided.
	3.	It is expected that all calculations and answers will be expressed as exact numbers such as 4π, $2+\sqrt{7}$, etc., rather than as $12.566\ldots$ or $4.646\ldots$.
	4.	**Calculators are permitted**, provided they are non-programmable and without graphic displays.
	5.	Diagrams are not drawn to scale. They are intended as aids only.

PART A

For each question in Part A, full marks will be given for a correct answer which is placed in the box. Part marks will be awarded only if relevant work is shown in the space provided in the answer booklet.

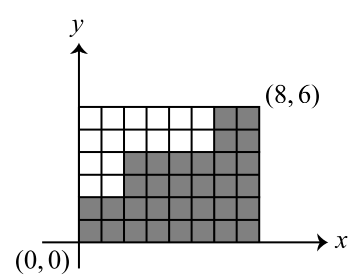
1. In the diagram, points B and C lie on AD. What is the value of x?



2. What is the smallest positive integer that is a multiple of each of 2, 4, 6, and 8?

3. Suppose that $y = ax + (1-a)$ for some unknown number a.
If $x = 3$, the value of y is 7. Then, if $x = 8$, what is the value of y?

4. An 8×6 grid is placed in the first quadrant with its edges along the axes, as shown. A total of 32 of the squares in the grid are shaded. A line is drawn through $(0,0)$ and $(8,c)$ cutting the shaded region into two equal areas. What is the value of c?



5. A *palindrome* is a positive integer that is the same when read forwards or backwards. For example, 1331 is a palindrome. Determine the number of palindromes between 1000 and 10 000 that are multiples of 6.

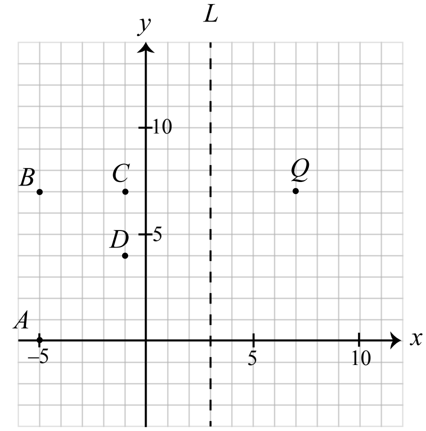
6. Note that $\frac{1}{2}+\frac{1}{3}+\frac{1}{6}=1$ and $\frac{1}{6}-\frac{1}{7}=\frac{1}{6\times 7}$ and $\frac{1}{2}+\frac{1}{3}+\frac{1}{7}+\frac{1}{42}=1$.

   Determine one triple (x,y,z) of positive integers with $1000 < x < y < z < 2000$ and

$$\frac{1}{2}+\frac{1}{3}+\frac{1}{7}+\frac{1}{x}+\frac{1}{y}+\frac{1}{z}+\frac{1}{45}=1$$

PART B

For each question in Part B, your solution must be well organized and contain words of explanation or justification when appropriate. Marks are awarded for completeness, clarity, and style of presentation. A correct solution, poorly presented, will not earn full marks.

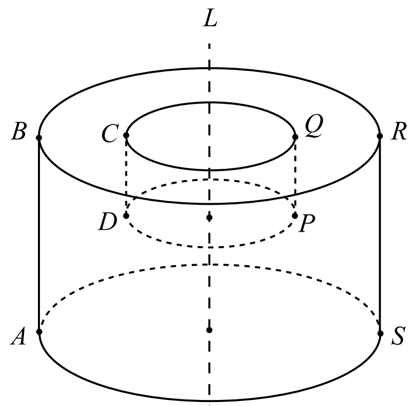


1. The points $A(-5,0)$, $B(-5,7)$, $C(-1,7)$, and $D(-1,4)$ are graphed, as shown. The line L has equation $x = 3$ and point Q is the reflection of point C in L.

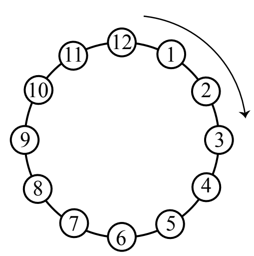
   (a) Point P is the reflection of point D in L. Point R is the reflection of point B in L. Point S is the reflection of point A in L. State the coordinates of P, R and S.

   (b) Calculate the perimeter of the figure $ABCDPQRS$.

   (c) Figure $ABCDPQRS$ is rotated around the line L to create a three-dimensional solid, as shown. This solid is a larger cylinder with a smaller cylinder removed. Determine the volume and the surface area of this solid. (Note: A cylinder with radius r and height h has volume $\pi r^2 h$ and surface area $2\pi r^2 + 2\pi rh$.)



2. In each part of this problem, cups are arranged in a circle and numbered $1, 2, 3$, and so on. A ball is placed in cup 1. Then, moving clockwise around the circle, a ball is placed in every nth cup. The process ends when cup 1 contains two balls. For example, starting with 12 cups and placing a ball in every 3rd cup, balls are placed, in order, in cups 1, 4, 7, 10, 1.



   (a) There are 12 cups in the circle and a ball is placed in every 5th cup, beginning and ending with cup 1. List, in order, the cups in which the balls are placed.

   (b) There are 9 cups in the circle and a ball is placed in every 6th cup, beginning and ending with cup 1. List the numbers of the cups that do not receive a ball.

   (c) There are 120 cups in the circle and a ball is placed in every 3rd cup, beginning and ending with cup 1. How many cups *do not* contain at least one ball when the process is complete? Explain how you obtained your answer.

   (d) There are 1000 cups in the circle and a ball is placed in every 7th cup, beginning and ending with cup 1. Determine the number of the cup into which the 338th ball is placed.

3. The *positive difference list (PDL)* of a set of integers is a list, written in increasing order, of the positive differences between all possible pairs of integers in the set. For example, the set $\{2, 5, 12\}$ produces a PDL consisting of three integers $3, 7, 10$ which are distinct, while the set $\{3, 4, 6, 9\}$ produces a PDL consisting of six integers $1, 2, 3, 3, 5, 6$ which are not distinct.

   (a) What is the PDL of the set of integers $\{3, 6, 13, 21, 32\}$?

   (b) Suppose that $x > 16$ and the sum of the integers in the PDL of the set $\{1, 4, 9, 16, x\}$ is 112. Determine the value of x.

   (c) State a set of integers of the form $\{3, q, r, s, 14\}$ with $3 < q < r < s < 14$ for which the PDL contains no repeated integers.

   (d) Prove that the PDL of every set of integers of the form $\{3, q, r, s, t\}$ with $3 < q < r < s < t$ and $t < 14$ contains repeated integers.