Fryer Contest

(Grade 9)

Thursday, April 12, 2012
(in North America and South America)

Friday, April 13, 2012
(outside of North America and South America)

UNIVERSITY OF
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Do not open this booklet until instructed to do so.

Time: 75 minutes   **Number of questions: 4**
Calculators are permitted   **Each question is worth 10 marks**

Parts of each question can be of two types:
1. **SHORT ANSWER** parts indicated by 💡

   - worth 2 or 3 marks each
   - full marks given for a correct answer which is placed in the box
   - **part marks awarded only if relevant work** is shown in the space provided

2. **FULL SOLUTION** parts indicated by ✒️

   - worth the remainder of the 10 marks for the question
   - **must be written in the appropriate location** in the answer booklet
   - marks awarded for completeness, clarity, and style of presentation
   - a correct solution poorly presented will not earn full marks

WRITE ALL ANSWERS IN THE ANSWER BOOKLET PROVIDED.
   - Extra paper for your finished solutions supplied by your supervising teacher must be inserted into your answer booklet. Write your name, school name, and question number on any inserted pages.
   - Express calculations and answers as exact numbers such as $\pi + 1$ and $\sqrt{2}$, etc., rather than as $4.14\ldots$ or $1.41\ldots$, except where otherwise indicated.

Do not discuss the problems or solutions from this contest online for the next 48 hours.

The name, grade, school and location of some top-scoring students will be published in the FGH Results on our Web site, http://www.cemc.uwaterloo.ca.

TIPS: 1.   Please read the instructions on the front cover of this booklet.
         2.   Write all answers in the answer booklet provided.

         3.   For questions marked 💡, place your answer in the appropriate box in the answer booklet and **show your work**.

         4.   For questions marked ✎, provide a well-organized solution in the answer booklet. Use mathematical statements and words to explain all of the steps of your solution. Work out some details in rough on a separate piece of paper before writing your finished solution.
         5.   Diagrams are *not* drawn to scale. They are intended as aids only.

1. 💡 (a) In Carrotford, candidate A ran for mayor and received 1008 votes out of a total of 5600 votes. What percentage of all votes did candidate A receive?

   💡 (b) In Beetland, exactly three candidates, B, C and D, ran for mayor. Candidate B won the election by receiving $\frac{3}{5}$ of all votes, while candidates C and D tied with the same number of votes. What percentage of all votes did candidate C receive?

   ✎ (c) In Cabbagetown, exactly two candidates, E and F, ran for mayor and 6000 votes were cast. At 10:00 p.m., only 90% of these votes had been counted. Candidate E received 53% of those votes. How many more votes had been counted for candidate E than for candidate F at 10:00 p.m.?

   ✎ (d) In Peaville, exactly three candidates, G, H and J, ran for mayor. When all of the votes were counted, G had received 2000 votes, H had received 40% of the votes, and J had received 35% of the votes. How many votes did candidate H receive?

2. The *prime factorization* of 144 is $2 \times 2 \times 2 \times 2 \times 3 \times 3$ or $2^4 \times 3^2$. Therefore, 144 is a perfect square because it can be written in the form $(2^2 \times 3) \times (2^2 \times 3)$.
   The prime factorization of 45 is $3^2 \times 5$. Therefore, 45 is not a perfect square, but 45×5 is a perfect square, because $45 \times 5 = 3^2 \times 5^2 = (3 \times 5) \times (3 \times 5)$.

   💡 (a) Determine the prime factorization of 112.

   💡 (b) The product $112 \times u$ is a perfect square. If u is a positive integer, what is the smallest possible value of u?

   ✎ (c) The product $5632 \times v$ is a perfect square. If v is a positive integer, what is the smallest possible value of v?

   ✎ (d) A *perfect cube* is an integer that can be written in the form n^3, where n is an integer. For example, 8 is a perfect cube since $8 = 2^3$. The product $112 \times w$ is a perfect cube. If w is a positive integer, what is the smallest possible value of w?

3. The positive integers are arranged in rows and columns, as shown, and described below.

	A	B	C	D	E	F	G
Row 1		1	2	3	4	5	6
Row 2	12	11	10	9	8	7	
Row 3		13	14	15	16	17	18
Row 4	24	23	22	21	20	19	

$$\vdots$$

The odd numbered rows list six positive integers in order from left to right beginning in column B. The even numbered rows list six positive integers in order from right to left beginning in column F.

(a) Determine the largest integer in row 30.

(b) Determine the sum of the six integers in row 2012.

(c) Determine the row and column in which the integer 5000 appears.

(d) For how many rows is the sum of the six integers in the row greater than $10\,000$ and less than $20\,000$?

4. The volume of a cylinder with radius r and height h equals $\pi r^2 h$.
The volume of a sphere with radius r equals $\frac{4}{3}\pi r^3$.

(a) The diagram shows a sphere that fits exactly inside a cylinder. That is, the top and bottom faces of the cylinder touch the sphere, and the cylinder and the sphere have the same radius, r. State an equation relating the height of the cylinder, h, to the radius of the sphere, r.
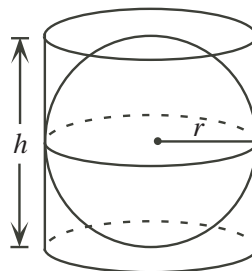
(b) For the cylinder and sphere given in part (a), determine the volume of the cylinder if the volume of the sphere is 288π.

(c) A solid cube with edges of length $1\,\mathrm{km}$ is fixed in outer space. Darla, the baby space ant, travels on this cube and in the space around (but not inside) this cube. If Darla is allowed to travel no farther than $1\,\mathrm{km}$ from the nearest point on the cube, then determine the total volume of space that Darla can occupy.

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In 2011, more than 13 000 students from around the world registered to write the Fryer, Galois and Hypatia Contests.

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