



The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING

www.cemc.uwaterloo.ca

Fermat Contest

(Grade 11)

Thursday, February 23, 2012
(in North America and South America)

Friday, February 24, 2012
(outside of North America and South America)

UNIVERSITY OF
WATERLOO

WATERLOO
MATHEMATICS

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Time: 60 minutes

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Calculators are permitted

Instructions

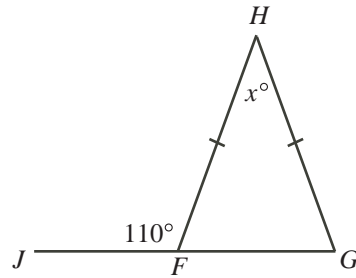
1. Do not open the Contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name and city/town in the box in the upper left corner.
5. **Be certain that you code your name, age, sex, grade, and the Contest you are writing in the response form. Only those who do so can be counted as eligible students.**
6. This is a multiple-choice test. Each question is followed by five possible answers marked **A, B, C, D,** and **E.** Only one of these is correct. After making your choice, fill in the appropriate circle on the response form.
7. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
There is *no penalty* for an incorrect answer.
Each unanswered question is worth 2, to a maximum of 10 unanswered questions.
8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor tells you to begin, you will have *sixty* minutes of working time.

The names of some top-scoring students will be published in the PCF Results on our Web site, <http://www.cemc.uwaterloo.ca>.

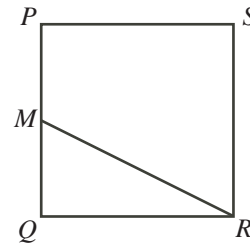
Scoring: There is *no penalty* for an incorrect answer.
 Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

Part A: Each correct answer is worth 5.

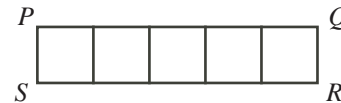
- Which of the following is *not equal* to a whole number?
 (A) $\frac{60}{12}$ (B) $\frac{60}{8}$ (C) $\frac{60}{5}$ (D) $\frac{60}{4}$ (E) $\frac{60}{3}$
- If $3 - 5 + 7 = 6 - x$, then x equals
 (A) -3 (B) -1 (C) 1 (D) 11 (E) 15
- In the diagram, $HF = HG$ and JFG is a straight line segment. The value of x is
 (A) 45 (B) 35 (C) 55
 (D) 60 (E) 40



- The value of $(1 + \frac{1}{3})(1 + \frac{1}{4})$ is
 (A) $\frac{5}{3}$ (B) $\frac{1}{3}$ (C) $\frac{13}{12}$ (D) $\frac{31}{12}$ (E) $\frac{16}{7}$
- In the diagram, $PQRS$ is a square and M is the midpoint of PQ . The area of triangle MQR is 100. The area of the square $PQRS$ is
 (A) 200 (B) 500 (C) 300
 (D) 400 (E) 800



- John ate a total of 120 peanuts over four consecutive nights. Each night he ate 6 more peanuts than the night before. How many peanuts did he eat on the fourth night?
 (A) 42 (B) 39 (C) 30 (D) 36 (E) 33
- Five identical squares form rectangle $PQRS$, as shown. The perimeter of rectangle $PQRS$ is 48. What is the area of $PQRS$?
 (A) 45 (B) 9 (C) 80
 (D) 16 (E) 96

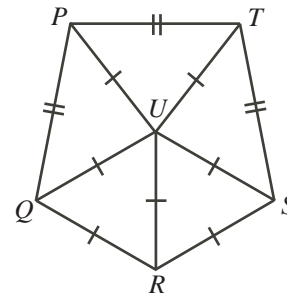


- If $x = 2$ and $v = 3x$, then the value of $(2v - 5) - (2x - 5)$ is
 (A) 2 (B) 8 (C) -2 (D) -7 (E) 6

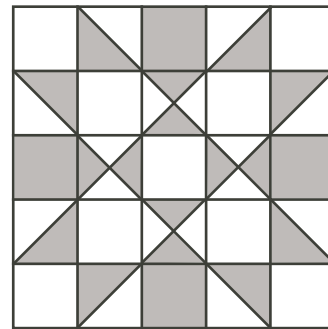
9. Mary and Sally were once the same height. Since then, Sally grew 20% taller and Mary's height increased by half as many centimetres as Sally's height increased. Sally is now 180 cm tall. How tall, in cm, is Mary now?
- (A) 144 (B) 165 (C) 162 (D) 150 (E) 170
10. If $(2^a)(2^b) = 64$, then the mean (average) of a and b is
- (A) 12 (B) $\frac{5}{2}$ (C) $\frac{3}{2}$ (D) 3 (E) 8

Part B: Each correct answer is worth 6.

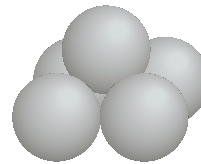
11. There is one odd integer N between 400 and 600 that is divisible by both 5 and 11. The sum of the digits of N is
- (A) 11 (B) 8 (C) 10 (D) 16 (E) 18
12. In the diagram, $\triangle QUR$ and $\triangle SUR$ are equilateral triangles. Also, $\triangle QUP$, $\triangle PUT$ and $\triangle TUS$ are isosceles triangles with $PQ = QU = UR = RS = ST = TP$ and $PU = QU = SU = TU$ and $QP = PT = TS$. The measure of $\angle UST$, in degrees, is
- (A) 50 (B) 54 (C) 60
(D) 70 (E) 80



13. The diagram shows a square quilt that is made up of identical squares and two sizes of right-angled isosceles triangles. What percentage of the quilt is shaded?
- (A) 36% (B) 40% (C) 44%
(D) 48% (E) 50%



14. The product of the roots of the equation $(x - 4)(x - 2) + (x - 2)(x - 6) = 0$ is
- (A) 20 (B) 48 (C) 10 (D) 96 (E) 2
15. Oranges are placed in a pyramid-like stack with each layer completely filled. The base is a rectangle that is 5 oranges wide and 7 oranges long. Each orange, above the first layer, rests in a pocket formed by four oranges in the layer below, as shown. The last layer is a single row of oranges. The total number of oranges in the stack is
- (A) 53 (B) 80 (C) 82
(D) 85 (E) 105



16. There are 30 people in a room, 60% of whom are men. If no men enter or leave the room, how many women must enter the room so that 40% of the total number of people in the room are men?

(A) 10 (B) 6 (C) 20 (D) 12 (E) 15

17. The expression $\frac{3^{2011} + 3^{2011}}{3^{2010} + 3^{2012}}$ is equal to

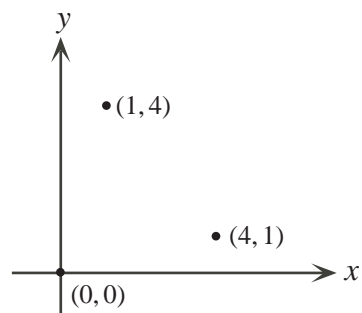
(A) $\frac{3}{5}$ (B) 1 (C) $\frac{9}{10}$ (D) $\frac{10}{3}$ (E) $\frac{2}{3}$

18. If N is the smallest positive integer whose digits have a product of 1728, then the sum of the digits of N is

(A) 28 (B) 26 (C) 18 (D) 27 (E) 21

19. The coordinates of three of the vertices of a parallelogram are $(0,0)$, $(1,4)$ and $(4,1)$. What is the area of this parallelogram?

(A) 15 (B) 19 (C) 16
(D) 17 (E) 12



20. Katie and Sarah run at different but constant speeds. They ran two races on a track that measured 100 m from start to finish. In the first race, when Katie crossed the finish line, Sarah was 5 m behind. In the second race, Katie started 5 m behind the original start line and they ran at the same speeds as in the first race. What was the outcome of the second race?

(A) Katie and Sarah crossed the finish line at the same time.
(B) When Katie crossed the finish line, Sarah was 0.25 m behind.
(C) When Katie crossed the finish line, Sarah was 0.26 m behind.
(D) When Sarah crossed the finish line, Katie was 0.25 m behind.
(E) When Sarah crossed the finish line, Katie was 0.26 m behind.

Part C: Each correct answer is worth 8.

21. If $x^2 = 8x + y$ and $y^2 = x + 8y$ with $x \neq y$, then the value of $x^2 + y^2$ is

(A) 9 (B) 49 (C) 63 (D) 21 (E) 56

22. In the country of Nohills, each pair of cities is connected by a straight (and flat) road. The chart to the right shows the distances along the straight roads between some pairs of cities. The distance along the straight road between city P and city R is closest to

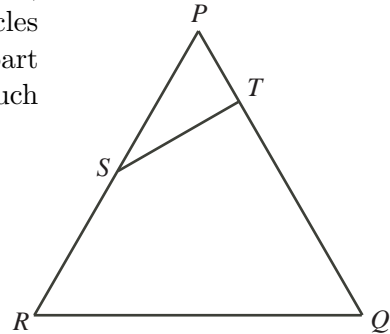
(A) 30 (B) 25 (C) 27
(D) 24.5 (E) 24

	P	Q	R	S
P	0	25		24
Q	25	0	25	7
R		25	0	18
S	24	7	18	0

23. A bowl contained 320 grams of pure white sugar. Mixture Y was formed by taking x grams of the white sugar out of the bowl, adding x grams of brown sugar to the bowl, and then mixing uniformly. In Mixture Y, the ratio of the mass of the white sugar to the mass of the brown sugar, expressed in lowest terms, was $w : b$. Mixture Z is formed by taking x grams of Mixture Y out of the bowl, adding x grams of brown sugar to the bowl, and then mixing uniformly. In Mixture Z, the ratio of the mass of the white sugar to the mass of the brown sugar is $49 : 15$. The value of $x + w + b$ is
- (A) 48 (B) 49 (C) 139 (D) 76 (E) 104

24. In equilateral triangle PQR , S is the midpoint of PR , T is on PQ so that $PT = 1$ and $TQ = 3$. Many circles can be drawn inside quadrilateral $QRST$ so that no part extends outside of $QRST$. The radius of the largest such circle is closest to

- (A) 1.00 (B) 1.10 (C) 1.15
 (D) 1.05 (E) 1.37



25. There are many positive integers N with the following properties:
- the digits of N include at least one of each of the digits 3, 4, 5, and 6,
 - the digits of N include no digits other than 3, 4, 5, and 6, and
 - the sum of the digits of N is 900 and the sum of the digits of $2N$ is 900.
- When the largest and smallest possible values of N are multiplied together, the number of digits in the resulting product is
- (A) 408 (B) 400 (C) 432 (D) 416 (E) 424



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