The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING
www.cemc.uwaterloo.ca

Canadian Senior Mathematics Contest

Tuesday, November 20, 2012
(in North America and South America)

Wednesday, November 21, 2012
(outside of North America and South America)

UNIVERSITY OF WATERLOO   WATERLOO MATHEMATICS

Time: 2 hours

Calculators are permitted, provided they are non-programmable and without graphic displays.

Do not open this booklet until instructed to do so.
There are two parts to this paper.

PART A

1. This part consists of six questions, each worth 5 marks.
2. **Enter the answer in the appropriate box in the answer booklet.**
   For these questions, full marks will be given for a correct answer which is placed in the box. Part marks will be awarded **only if relevant work** is shown in the space provided in the answer booklet.

PART B

1. This part consists of three questions, each worth 10 marks.
2. **Finished solutions must be written in the appropriate location in the answer booklet.** Rough work should be done separately. If you require extra pages for your finished solutions, they will be supplied by your supervising teacher. Insert these pages into your answer booklet. Be sure to write your name, school name and question number on any inserted pages.
3. Marks are awarded for completeness, clarity, and style of presentation. A correct solution, poorly presented, will not earn full marks.

NOTES:
The questions in each part are arranged roughly in order of increasing difficulty.
The early problems in Part B are likely easier than the later problems in Part A.
At the completion of the contest, insert your student information form inside your answer booklet.

Do not discuss the problems or solutions from this contest online for the next 48 hours.

The name, grade, school and location, and score range of some top-scoring students will be published on the Web site, http://www.cemc.uwaterloo.ca. In addition, the name, grade, school and location, and score of some students may be shared with other mathematical organizations for other recognition opportunities.

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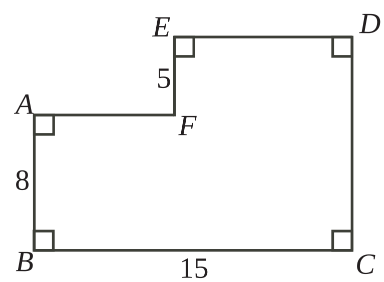
NOTE:	1.	Please read the instructions on the front cover of this booklet.
	2.	Write solutions in the answer booklet provided.
	3.	It is expected that all calculations and answers will be expressed as exact numbers such as 4π, $2+\sqrt{7}$, etc., rather than as $12.566\ldots$ or $4.646\ldots$.
	4.	**Calculators are permitted**, provided they are non-programmable and without graphic displays.
	5.	Diagrams are not drawn to scale. They are intended as aids only.

PART A

For each question in Part A, full marks will be given for a correct answer which is placed in the box. Part marks will be awarded only if relevant work is shown in the space provided in the answer booklet.

1. Figure $ABCDEF$ has $AB = 8$, $BC = 15$, and $EF = 5$, as shown. Determine the perimeter of $ABCDEF$.

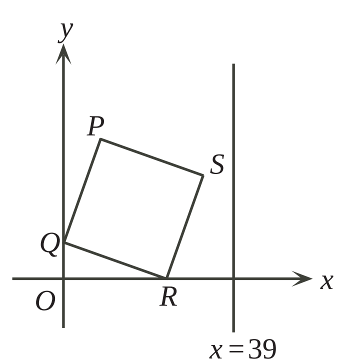


2. There are three distinct real numbers a, b and c that are solutions of the equation $x^3 - 4x = 0$. What is the value of the product abc?

3. If $3^x = 3^{20} \cdot 3^{20} \cdot 3^{18} + 3^{19} \cdot 3^{20} \cdot 3^{19} + 3^{18} \cdot 3^{21} \cdot 3^{19}$, determine the value of x.

4. Three boxes each contain an equal number of hockey pucks. Each puck is either black or gold. All 40 of the black pucks and exactly $\frac{1}{7}$ of the gold pucks are contained in one of the three boxes. Determine the total number of gold hockey pucks.

5. In the diagram, square $PQRS$ has side length 25, Q is located at $(0, 7)$, and R is on the x-axis. The square is rotated clockwise about R until S lies above the x-axis on the line with equation $x = 39$. What are the new coordinates of P after this rotation?

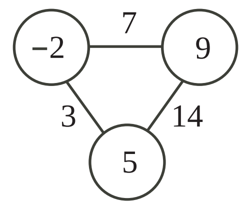


6. Lynne is tiling her long and narrow rectangular front hall. The hall is 2 tiles wide and 13 tiles long. She is going to use exactly 11 black tiles and exactly 15 white tiles. Determine the number of distinct ways of tiling the hall so that no two black tiles are adjacent (that is, share an edge).

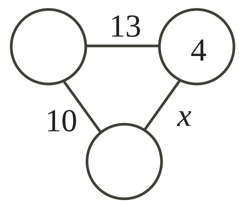
PART B

For each question in Part B, your solution must be well organized and contain words of explanation or justification when appropriate. Marks are awarded for completeness, clarity, and style of presentation. A correct solution, poorly presented, will not earn full marks.

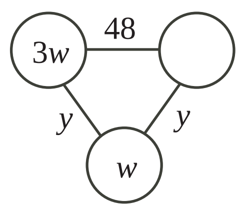
1. In each diagram shown in this problem, the number on the line connecting two circles is the sum of the two numbers in these two circles. An example of a completed diagram is shown to the right.



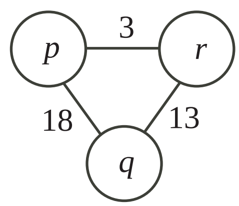
   (a) What is the value of x?



   (b) With justification, determine the value of y.



   (c) With justification, determine the values of p, q and r.



2. Consider the equation $x^2 - 2y^2 = 1$, which we label ①. There are many pairs of positive integers (x, y) that satisfy equation ①.

   (a) Determine a pair of positive integers (x, y) with $x \leq 5$ that satisfies equation ①.

   (b) Determine a pair of positive integers (u, v) such that

$$(3 + 2\sqrt{2})^2 = u + v\sqrt{2}$$

   and show that (u, v) satisfies equation ①.

   (c) Suppose that (a, b) is a pair of positive integers that satisfies equation ①. Suppose also that (c, d) is a pair of positive integers such that

$$(a + b\sqrt{2})(3 + 2\sqrt{2}) = c + d\sqrt{2}$$

   Show that (c, d) satisfies equation ①.

   (d) Determine a pair of positive integers (x, y) with $y > 100$ that satisfies equation ①.

3. (a) Right-angled $\triangle PQR$ has $\angle PQR = 90°$, $PQ = 6$ and $QR = 8$. If M is the midpoint of QR and N is the midpoint of PQ, determine the lengths of the medians PM and RN.

   (b) $\triangle DEF$ has two medians of equal length. Prove that $\triangle DEF$ is isosceles.

   (c) $\triangle ABC$ has its vertices on a circle of radius r. If the lengths of two of the medians of $\triangle ABC$ are equal to r, determine the side lengths of $\triangle ABC$.