Canadian Intermediate Mathematics Contest

Tuesday, November 22, 2011
(in North America and South America)

Wednesday, November 23, 2011
(outside of North America and South America)

UNIVERSITY OF WATERLOO

WATERLOO MATHEMATICS

Time: 2 hours   ©2011 University of Waterloo

Calculators are permitted, provided they are non-programmable and without graphic displays.

Do not open this booklet until instructed to do so.

There are two parts to this paper.

PART A

1. This part consists of six questions, each worth 5 marks.

2. **Enter the answer in the appropriate box in the answer booklet.**
   For these questions, full marks will be given for a correct answer which is placed in the box. Part marks will be awarded **only if relevant work** is shown in the space provided in the answer booklet.

PART B

1. This part consists of three questions, each worth 10 marks.

2. **Finished solutions must be written in the appropriate location in the answer booklet.** Rough work should be done separately. If you require extra pages for your finished solutions, they will be supplied by your supervising teacher. Insert these pages into your answer booklet. Be sure to write your name, school name and question number on any inserted pages.

3. Marks are awarded for completeness, clarity, and style of presentation. A correct solution, poorly presented, will not earn full marks.

NOTES:

At the completion of the contest, insert your student information form inside your answer booklet.

The names of some top-scoring students will be published on the CEMC website, http://www.cemc.uwaterloo.ca.

Canadian Intermediate Mathematics Contest

| NOTE:	1.	Please read the instructions on the front cover of this booklet.
	2.	Write solutions in the answer booklet provided.
	3.	It is expected that all calculations and answers will be expressed as exact numbers such as 4π, $2 + \sqrt{7}$, etc., rather than as $12.566\ldots$ or $4.646\ldots$.
	4.	**Calculators are permitted**, provided they are non-programmable and without graphic displays.
	5.	Diagrams are not drawn to scale. They are intended as aids only.

Useful Fact:
It may be helpful to know that if n is a positive integer, then the sum of the n integers from 1 to n equals $\frac{1}{2}n(n+1)$; that is, $1 + 2 + 3 + \cdots + (n-1) + n = \frac{1}{2}n(n+1)$.

PART A

For each question in Part A, full marks will be given for a correct answer which is placed in the box. Part marks will be awarded only if relevant work is shown in the space provided in the answer booklet.

1. Joey is standing at a corner of the rectangular field shown. He walks the perimeter of the field 5 times. How many metres does Joey walk?

   Joey   100 m   50 m
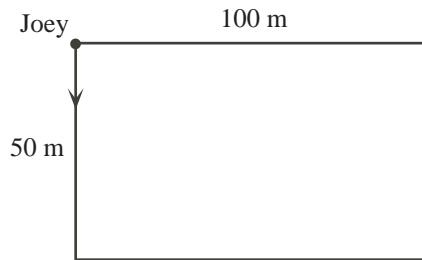
2. If $a + b = 9 - c$ and $a + b = 5 + c$, what is the value of c?

3. Ophelia is paid \$51 for the first week of her part-time summer job. For each week after the first, she is paid \$100. How many weeks in total does she have to work for her average weekly pay to be \$93?

4. A die is a cube with its faces numbered 1 through 6. One red die and one blue die are rolled. The sum of the numbers on the top face of each die is determined. What is the probability that this sum is a perfect square?

5. In the diagram, point P is inside quadrilateral $ABCD$. Also, $DA = DP = DC$ and $AP = AB$. If $\angle ADP = \angle CDP = 2x°$, $\angle BAP = (x + 5)°$, and $\angle BPC = (10x - 5)°$, what is the value of x?
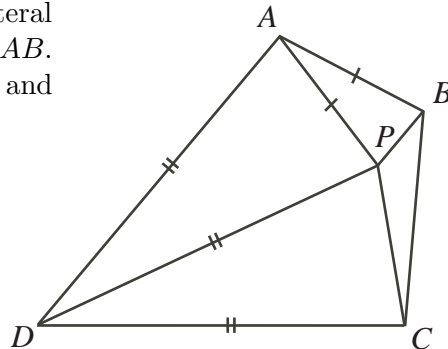
6. The product of all of the positive integer divisors of 6^{16} equals 6^k for some integer k. Determine the value of k.

PART B

For each question in Part B, your solution must be well organized and contain words of explanation or justification when appropriate. Marks are awarded for completeness, clarity, and style of presentation. A correct solution, poorly presented, will not earn full marks.

1. In each of Figure A, Figure B, and Figure D, the circle has centre O and radius 2.

   (a) Determine the area of the shaded region in Figure A.

   (b) Determine the area of the shaded region in Figure B.

   (c) In Figure C, $\triangle PQR$ is equilateral and has side length of 2. Determine the area of $\triangle PQR$.

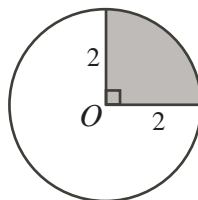   (d) Determine the area of the shaded region in Figure D.
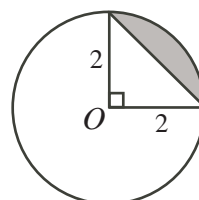

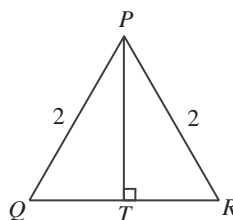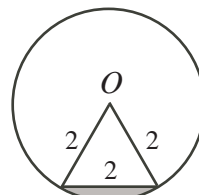Figure A   Figure B

Figure C   Figure D

2. A pattern of Figures is shown below. Figure 1 is a regular pentagon with side length 1. Figure 2 is a regular pentagon of side length 2 drawn around Figure 1 so that the two shapes share the top vertex, T, and the sides on either side of T overlap. The pattern continues so that for each $n > 1$, Figure n is a regular pentagon of side length n drawn around the previous Figure so that the two shapes share the top vertex, T, and the sides on either side of T overlap. The *ink length* of each Figure is the sum of the lengths of all of the line segments in the Figure.
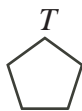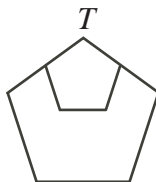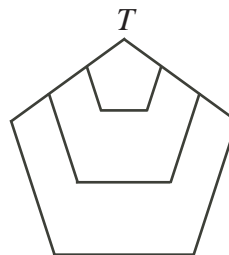

Figure 1
Ink length: 5

Figure 2
Ink length: 13

Figure 3
Ink length: 24

   (a) Determine the ink length of Figure 4.

   (b) Determine the difference between the ink length of Figure 9 and the ink length of Figure 8.

   (c) Determine the ink length of Figure 100.

2011
Canadian
Intermediate
Mathematics
Contest
(English)

3. Six swimmers swim lengths in a pool that is 100 m long. They start at the same time from the south end (S) of the pool, swim to the north end (N), swim back to S, then to N, and so on. They each swim at a constant speed and each turns around instantly at both ends of the pool. The swimmers are said to *cross* when they pass each other in the pool *while swimming in opposite directions*. We also say that they cross if they arrive at N or at S at the same time.

   (a) Suppose that Alice and Betty cross at S after Alice has swum 400 m and Betty has swum 600 m. How many times *before* this point did they cross?

   (b) Suppose that Charlie and David first cross when Charlie has swum 90 m. How many times will they cross again *before* arriving at S together for the first time?

   (c) Suppose that Farouq swims faster than Evan. Show that the length of time between two consecutive instances when Evan and Farouq cross is constant.