2010 Hypatia Contest (Grade 11)

Friday, April 9, 2010

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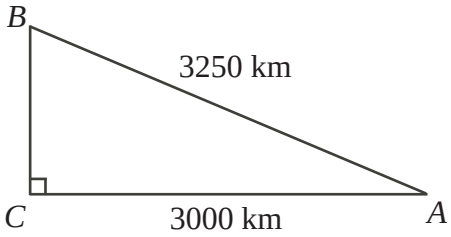
1. Piravena must make a trip from A to B, then from B to C, then from C to A. Each of these three parts of the trip is made entirely by bus or entirely by airplane. The cities form a right-angled triangle as shown, with C a distance of 3000 km from A and with B a distance of 3250 km from A. To take a bus, it costs Piravena \$0.15 per kilometre. To take an airplane, it costs her a \$100 booking fee, plus \$0.10 per kilometre.



   (a) To begin her trip she flew from A to B. Determine the cost to fly from A to B.

   (b) Determine the distance she travels for her complete trip.

   (c) Piravena chose the least expensive way to travel between cities and her total cost was \$1012.50. Given that she flew from A to B, determine her method of transportation from B to C and her method of transportation from C to A.

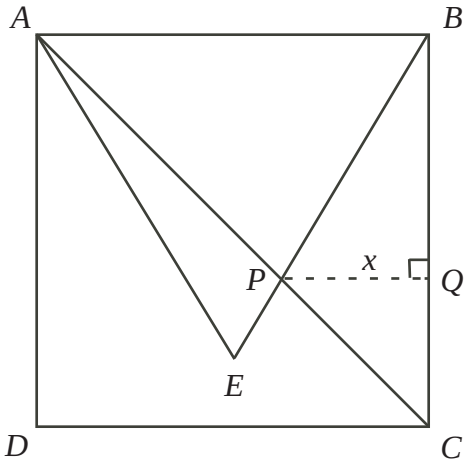
2. A function f is such that $f(x) - f(x-1) = 4x - 9$ and $f(5) = 18$.

   (a) Determine the value of $f(6)$.

   (b) Determine the value of $f(3)$.

   (c) If $f(x) = 2x^2 + px + q$, determine the values of p and q.

3. In the diagram, square $ABCD$ has sides of length 4, and $\triangle ABE$ is equilateral. Line segments BE and AC intersect at P. Point Q is on BC so that PQ is perpendicular to BC and $PQ = x$.



   (a) Determine the measures of the angles of $\triangle BPC$.

   (b) Find an expression for the length of BQ in terms of x.

   (c) Determine the exact value of x.

   (d) Determine the exact area of $\triangle APE$.

4. (a) Determine all real values of x satisfying the equation $x^4 - 6x^2 + 8 = 0$.

   (b) Determine the smallest positive integer N for which $x^4 + 2010x^2 + N$ can be factored as $(x^2 + rx + s)(x^2 + tx + u)$ with r, s, t, u integers and $r \neq 0$.

   (c) Prove that $x^4 + Mx^2 + N$ cannot be factored as in (b) for any integers M and N with $N - M = 37$.