Pascal Contest *(Grade 9)*

Wednesday, February 18, 2009

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Instructions

1. Do not open the Contest booklet until you are told to do so.

2. You may use rulers, compasses and paper for rough work.

3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.

4. On your response form, print your school name, city/town, and province in the box in the upper left corner.

5. **Be certain that you code your name, age, sex, grade, and the Contest you are writing in the response form. Only those who do so can be counted as official contestants.**

6. This is a multiple-choice test. Each question is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. After making your choice, fill in the appropriate circle on the response form.

7. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
   There is *no penalty* for an incorrect answer.
   Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

8. Diagrams are *not* drawn to scale. They are intended as aids only.

9. When your supervisor tells you to begin, you will have *sixty* minutes of working time.

The names of some top-scoring students will be published in the PCF Results on our Web site, http://www.cemc.uwaterloo.ca.

Part A: Each correct answer is worth 5.

1. What is the value of $2 \times 9 - \sqrt{36} + 1$?

   (A) 7   **(B)** 11   **(C)** 8   **(D)** 13   **(E)** 4

2. The graph shows the number of hours Deepit worked over a three day period. What is the total number of hours that he worked on Saturday and Sunday?

   (A) 2   **(B)** 4   **(C)** 6

   (D) 8   **(E)** 10
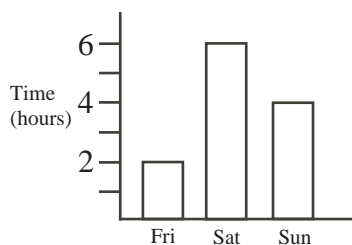
3. The cost of 1 piece of gum is 1 cent. What is the cost of 1000 pieces of gum?
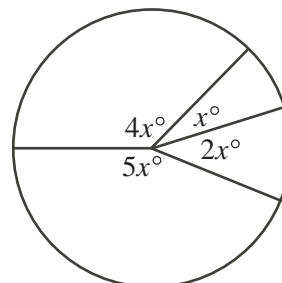
   (A) \$0.01   **(B)** \$0.10   **(C)** \$1.00   **(D)** \$10.00   **(E)** \$100.00

4. There are 18 classes at Webster Middle School. Each class has 28 students. On Monday, 496 students were at school. How many students were absent?

   (A) 8   **(B)** 11   **(C)** 18   **(D)** 26   **(E)** 29

5. In the diagram, the value of x is

   (A) 15   **(B)** 20   **(C)** 24

   (D) 30   **(E)** 36

6. What is the value of $(-1)^5 - (-1)^4$?

   (A) -2 **(B)** -1 **(C)** 0   **(D)** 1   **(E)** 2

7. In the diagram, $\triangle PQR$ is right-angled at Q, PQ is horizontal and QR is vertical. What are the coordinates of Q?
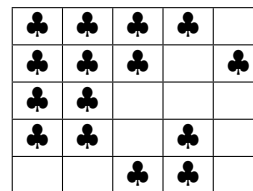
   (A) $(5,2)$ **(B)** $(5,0)$ **(C)** $(5,1)$

   (D) $(4,1)$ **(E)** $(1,5)$
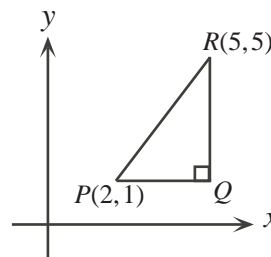
8. If $y = 3$, the value of $\dfrac{y^3 + y}{y^2 - y}$ is

   (A) 2   **(B)** 3   **(C)** 4   **(D)** 5   **(E)** 6

9. In the diagram, any ♣ may be moved to any unoccupied space. What is the smallest number of ♣'s that must be moved so that each row and each column contains three ♣'s?

(A) 1   (B) 2   (C) 3

(D) 4   (E) 5

10. If $z = 4$, $x + y = 7$, and $x + z = 8$, what is the value of $x + y + z$?

(A) 9   (B) 17   (C) 11   (D) 19   (E) 13

Part B: Each correct answer is worth 6.

11. When the numbers $5.07\overline{6}$, $5.0\overline{76}$, 5.07, 5.076, $5.\overline{076}$ are arranged in increasing order, the number in the middle is
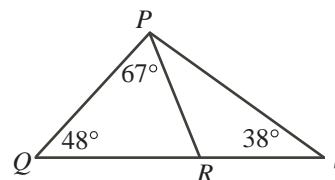
(A) $5.07\overline{6}$ (B) $5.0\overline{76}$ (C) 5.07 (D) 5.076 (E) $5.\overline{076}$

12. If Francis spends $\frac{1}{3}$ of his day sleeping, $\frac{1}{4}$ of his day studying and $\frac{1}{8}$ of his day eating, how many hours in the day does he have left?
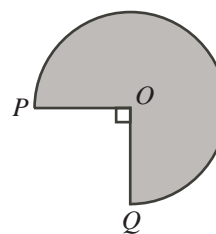
(A) 4   (B) 6   (C) 5   (D) 7   (E) 9

13. In the diagram, QRS is a straight line. What is the measure of $\angle RPS$?

(A) $27°$ (B) $47°$ (C) $48°$

(D) $65°$ (E) $67°$

14. In the diagram, O is the centre of a circle with radii $OP = OQ = 5$. The *perimeter* of the shaded region, including the two radii, is closest to

(A) 34   (B) 41   (C) 52

(D) 59   (E) 68

15. The increasing list of five different integers $\{3, 4, 5, 8, 9\}$ has a sum of 29. How many increasing lists of five different single-digit positive integers have a sum of 33?

(A) 1   (B) 2   (C) 3   (D) 4   (E) 5

16. In the diagram, a 4×9 grid $PQTV$ is formed from thirty-six 1×1 squares. Lines PR and US are drawn with R and S on QT. What is the ratio of the shaded area to the unshaded area?

(A) $5 : 9$ (B) $9 : 8$ (C) $4 : 5$

(D) $9 : 5$ (E) $5 : 4$
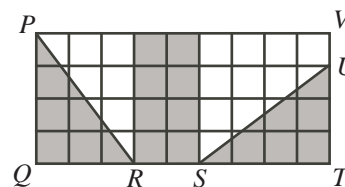
17. Nerissa writes five mathematics tests, each worth the same amount, and obtains an average of 73%. After her teacher deletes one of her test marks, Nerissa's new average is 76%. What was the mark on the test that the teacher deleted?

(A) 60%   (B) 61%   (C) 62%   (D) 63%   (E) 64%

18. Every 4 years, the population of the town of Arloe doubles. On December 31, 2008, the population of Arloe was 3456. What was the population on December 31, 1988?

(A) 54   (B) 576   (C) 216   (D) 108   (E) 864

19. The distance from Coe Hill to Calabogie is 150 kilometres. Pat leaves Coe Hill at 1:00 p.m. and drives at a speed of 80 km/h for the first 60 km. How fast must he travel for the remainder of the trip to reach Calabogie at 3:00 p.m.?

(A) 65 km/h   (B) 70 km/h   (C) 72 km/h   (D) 75 km/h   (E) 90 km/h

20. Different positive integers can be written in the eight empty circles so that the product of any three integers in a straight line is 3240. What is the largest possible sum of the eight numbers surrounding 45?

(A) 139   (B) 211   (C) 156

(D) 159   (E) 160
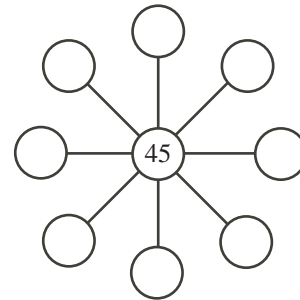
Part C: Each correct answer is worth 8.

21. Alice rolls a standard 6-sided die. Bob rolls a second standard 6-sided die. Alice wins if the values shown differ by 1. What is the probability that Alice wins?

(A) $\frac{1}{3}$ (B) $\frac{2}{9}$ (C) $\frac{5}{18}$ (D) $\frac{1}{6}$ (E) $\frac{5}{36}$

22. In the diagram, PQ and RS are diameters of a circle with radius 4. If PQ and RS are perpendicular, what is the area of the shaded region?

(A) $16 + 4\pi$ (B) $8 + 8\pi$ (C) $8 + 4\pi$

(D) $16 + 16\pi$ (E) $16 + 8\pi$

23. A one-dollar coin should have a mass of 7.0 g. Each individual coin may be lighter or heavier by as much as 2.14%. Joshua has a number of these coins and determines that they have a total mass of 1 kg. What is the difference between the greatest possible number and the least possible number of these coins that he could have?

(A) 4   (B) 5   (C) 6   (D) 7   (E) 8
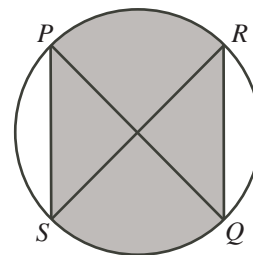
24. Eight identical spheres, each of diameter 20, fit tightly into a cube of side length 40 so that each sphere just touches three of the faces of the cube. The radius of the largest sphere that will fit in the central space, just touching all eight spheres, is closest to

(A) 7.0   (B) 7.3   (C) 7.6   (D) 7.9   (E) 8.2

25. Starting with the input (m, n), Machine A gives the output (n, m).
Starting with the input (m, n), Machine B gives the output $(m + 3n, n)$.
Starting with the input (m, n), Machine C gives the output $(m - 2n, n)$.
Natalie starts with the pair $(0, 1)$ and inputs it into one of the machines. She takes the output and inputs it into any one of the machines. She continues to take the output that she receives and inputs it into any one of the machines. (For example, starting with $(0, 1)$, she could use machines B, B, A, C, B in that order to obtain the output $(7, 6)$.) Which of the following pairs is impossible for her to obtain after repeating this process any number of times?

(A) (2009,1016)   (B) (2009,1004)   (C) (2009,1002)
(D) (2009,1008)   (E) (2009,1032)

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