# 2009 Hypatia Contest (Grade 11)
Wednesday, April 8, 2009

1. Emma counts the number of students in her class with each eye and hair colour, and summarizes the results in the following table:

Hair Colour

		Brown	Blonde	Red
	Blue	3	2	1
Eye Colour	Green	2	4	2
	Brown	2	3	1

   (a) What percentage of the students have both green eyes and brown hair?

   (b) What percentage of the students have green eyes or brown hair or both?

   (c) Of the students who have green eyes, what percentage also have red hair?

   (d) Determine how many students with red hair must join the class so that the percentage of the students in the class with red hair becomes 36%.

2. An arithmetic sequence is a sequence in which each term after the first is obtained from the previous term by adding a constant d, called the common difference. For example, the sequence 2, 11, 20, 29, 38 is an arithmetic sequence with five terms and a common difference of $d = 9$.

   (a) An arithmetic sequence has three terms. The three terms add to 180. Determine the middle term of this sequence.

   (b) An arithmetic sequence has five terms. The five terms add to 180. Show that at least one of the five terms equals 36.

   (c) An arithmetic sequence has six terms. The six terms in the sequence add to 180. Determine the sum of the first and sixth terms of the sequence.

3. Triangle ABC has vertices $A(0, 8)$, $B(2, 0)$, $C(8, 0)$.

   (a) Determine the equation of the line through B that cuts the area of $\triangle ABC$ in half.

   (b) A vertical line intersects AC at R and BC at S, forming $\triangle RSC$. If the area of $\triangle RSC$ is 12.5, determine the coordinates of point R.

   (c) A horizontal line intersects AB at T and AC at U, forming $\triangle ATU$. If the area of $\triangle ATU$ is 13.5, determine the equation of the horizontal line.

4. (a) A solid right prism $ABCDEF$ has a height of 16, as shown. Also, its bases are equilateral triangles with side length 12. Points X, Y, and Z are the midpoints of edges AC, BC, and DC, respectively. Determine the lengths of XY, YZ and XZ.
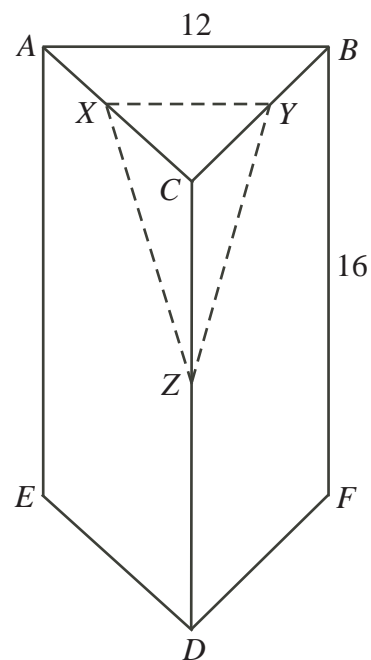
   (b) A part of the prism above is sliced off with a straight cut through points X, Y and Z. Determine the surface area of solid $CXYZ$, the part that was sliced off.

   (c) The prism $ABCDEF$ in part (a) is sliced with a straight cut through points M, N, P, and Q on edges DE, DF, CB, and CA, respectively. If $DM = 4$, $DN = 2$, and $CQ = 8$, determine the volume of the solid $QPCDMN$.