Canadian
Mathematics
Competition
An activity of the Centre for Education
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Grade 8 solutions follow the Grade 7 solutions

2009 Gauss Contests

(Grades 7 and 8)

Wednesday, May 13, 2009

Solutions

Centre for Education in Mathematics and Computing Faculty and Staff

Gauss Contest Committee

Grade 7

1. Adding, $4.1 + 1.05 + 2.005 = 5.15 + 2.005 = 7.155$.

Answer: (A)

2. Since the triangle is equilateral, all sides are equal in length.
Therefore, the perimeter of the triangle is $8 + 8 + 8 = 8 \times 3 = 24$.

Answer: (C)

3. The numbers 12, 14 and 16 are even, and therefore divisible by 2 so not prime.
The number 15 is divisible by 5; therefore, it is also not prime.
Each of the remaining numbers, 11, 13 and 17, has no positive divisor other than 1 and itself.
Therefore, 3 numbers in the list are prime.

Answer: (D)

4. *Solution 1*
Since each number in the set is between 0 and 1, they can be ordered from smallest to largest by comparing their tenths digits first. In order from smallest to largest the list is

$$\{0.05, 0.25, 0.37, 0.40, 0.81\}.$$

The smallest number in the list is 0.05.

Solution 2
Consider the equivalent fraction for each decimal:
$0.40 = \frac{40}{100}$, $0.25 = \frac{25}{100}$, $0.37 = \frac{37}{100}$, $0.05 = \frac{5}{100}$, and $0.81 = \frac{81}{100}$.
Since the denominators all equal 100, we choose the fraction with the smallest numerator. Therefore, $0.05 = \frac{5}{100}$ is the smallest number in the set.

Answer: (D)

5. The x-coordinate of point P lies between -2 and 0. The y-coordinate lies between 2 and 4.
Of the possible choices, $(-1, 3)$ is the only point that satisfies both of these conditions.

Answer: (E)

6. The temperature in Vancouver is 22°C.
The temperature in Calgary is $22°\text{C} - 19°\text{C} = 3°\text{C}$.
The temperature in Quebec City is $3°\text{C} - 11°\text{C} = -8°\text{C}$.

Answer: (C)

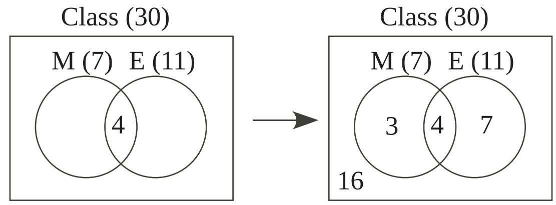
7. Since a real distance of 60 km is represented by 1 cm on the map, then a real distance of 540 km is represented by $\dfrac{540}{60}$ cm or 9 cm on the map.

Answer: (A)

8. The sum of the three angles in any triangle is always 180°.
In $\triangle PQR$, the sum of $\angle P$ and $\angle Q$ is 60°, and thus $\angle R$ must measure $180° - 60° = 120°$.

Answer: (C)

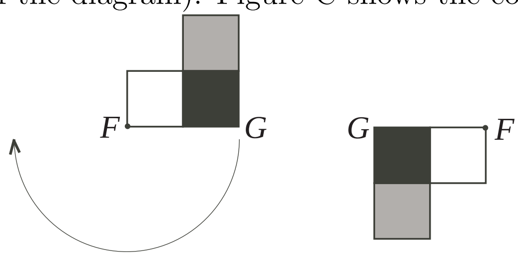
9. The first Venn diagram below shows that there are 30 students in the class, 7 students have been to Mexico, 11 students have been to England and 4 students have been to both countries. Of the 7 students that have been to Mexico, 4 have also been to England.
Therefore, $7 - 4 = 3$ students have been to Mexico and not England.
Of the 11 students that have been to England, 4 have also been to Mexico.
Therefore, $11 - 4 = 7$ students have been to England and not Mexico.



Therefore, 3 students have been to Mexico only, 7 students have been to England only, and 4 students have been to both.
In the class of 30 students, this leaves $30 - 3 - 7 - 4 = 16$ students who have not been to Mexico or England.

Answer: (B)

10. Consider rotating the horizontal line segment FG (as shown below) 180° about point F.
A 180° rotation is half of a full rotation. Point F stays fixed, while segment FG rotates to the left of F (as does the rest of the diagram). Figure C shows the correct result.



Answer: (C)

11. Since Scott runs 4 m for every 5 m Chris runs, Scott runs $\frac{4}{5}$ of the distance that Chris runs in the same time. When Chris crosses the finish line he will have run 100 m.
When Chris has run 100 m, Scott will have run $\frac{4}{5} \times 100 = 80$ m.

Answer: (E)

12. The area of a triangle can be calculated using the formula Area $= \frac{1}{2} \times$ base $\times$ height.
The area is 27 cm^2 and the base measures 6 cm. Substituting these values into the formula, $A = \frac{1}{2} \times b \times h$ becomes $27 = \frac{1}{2} \times 6 \times h$ or $27 = 3h$. Therefore, $h = 9$ cm.

Answer: (A)

13. There are 60 seconds in a minute, 60 minutes in an hour, 24 hours in a day and 7 days in a week. Therefore, the number of seconds in one week is $60 \times 60 \times 24 \times 7$.

Answer: (D)

14. *Solution 1*
S represents a value of approximately 1.5 on the number line, while T is approximately 1.6. Then $S \div T$ is approximately equal to $1.5 \div 1.6 = 0.9375$. R is the only value on the number line that is slightly less than 1 and therefore best represents the value of $S \div T$.

Solution 2
S is slightly less than T, so $\dfrac{S}{T}$ is slightly less than 1. Thus, $\dfrac{S}{T}$ is best represented by R.

Answer: (C)

15. For the sum to be a maximum, we try to use the largest divisor possible.
Although 144 is the largest divisor, using it would require that the remaining two divisors both equal 1 (since the divisors are integers).
Since the question requires the product of three different divisors, $144 = 144 \times 1 \times 1$ is not possible and the answer cannot be $144 + 1 + 1 = 146$ or (C).
The next largest divisor of 144 is 72 and $144 = 72 \times 2 \times 1$.
Now the three factors are different and their sum is $72 + 2 + 1 = 75$.
Since 75 is the largest possible answer remaining, we have found the maximum.

Answer: (B)

16. *Solution 1*
For the square to have an area of 25, each side length must be $\sqrt{25} = 5$.
The rectangle's width is equal to that of the square and therefore must also be 5.
The length of the rectangle is double its width or $5 \times 2 = 10$.
The area of the rectangle is thus $5 \times 10 = 50$.

Solution 2
The rectangle has the same width as the square but twice the length.
Thus, the rectangle's area is twice that of the square or $2 \times 25 = 50$.

Answer: (D)

17. The six other players on the team averaged 3.5 points each.
The total of their points was $6 \times 3.5 = 21$.
Vanessa scored the remainder of the points, or $48 - 21 = 27$ points.

Answer: (E)

18. Since x and z are positive integers and $xz = 3$, the only possibilities are $x = 1$ and $z = 3$ or $x = 3$ and $z = 1$.
Assuming that $x = 1$ and $z = 3$, $yz = 6$ implies $3y = 6$ or $y = 2$.
Thus, $x = 1$ and $y = 2$ and $xy = 2$.
This contradicts the first equation $xy = 18$.
Therefore, our assumption was incorrect and it must be true that $x = 3$ and $z = 1$.
Then $yz = 6$ and $z = 1$ implies $y = 6$.
Checking, $x = 3$ and $y = 6$ also satisfies $xy = 18$, the first equation.
Therefore, the required sum is $x + y + z = 3 + 6 + 1 = 10$.

Answer: (B)

19. The value of all quarters is \$10.00.
Each quarter has a value of \$0.25.
There are thus $10 \div 0.25 = 40$ quarters in the jar.
Similarly, there are $10 \div 0.05 = 200$ nickels, and $10 \div 0.01 = 1000$ pennies in the jar.
In total, there are $40 + 200 + 1000 = 1240$ coins in the jar.
The probability that the selected coin is a quarter is $\dfrac{\text{the number of quarters}}{\text{the total number of coins}} = \dfrac{40}{1240} = \dfrac{1}{31}$.

Answer: (B)

20. Since V is the midpoint of PR, then $PV = VR$.
Since $UVRW$ is a parallelogram, then $VR = UW$.
Since W is the midpoint of US, then $UW = WS$.
Thus, $PV = VR = UW = WS$.

Similarly, $QW = WR = UV = VT$.
Also, R is the midpoint of TS and therefore, $TR = RS$.
Thus, $\triangle VTR$ is congruent to $\triangle WRS$, and so the two triangles have equal area.
Diagonal VW in parallelogram $UVRW$ divides the area of the parallelogram in half.
Therefore, $\triangle UVW$ and $\triangle RWV$ have equal areas.
In quadrilateral $VRSW$, $VR = WS$ and VR is parallel to WS.
Thus, $VRSW$ is a parallelogram and the area of $\triangle RWV$ is equal to the area of $\triangle WRS$.
Therefore, $\triangle VTR$, $\triangle WRS$, $\triangle RWV$, and $\triangle UVW$ have equal areas, and so these four triangles divide $\triangle STU$ into quarters.
Parallelogram $UVRW$ is made from two of these four quarters of $\triangle STU$, or one half of $\triangle STU$.
The area of parallelogram $UVRW$ is thus $\frac{1}{2}$ of 1, or $\frac{1}{2}$.

Answer: (B)

21. Together, Lara and Ryan ate $\frac{1}{4} + \frac{3}{10} = \frac{5}{20} + \frac{6}{20} = \frac{11}{20}$ of the pie.
Therefore, $1 - \frac{11}{20} = \frac{9}{20}$ of the pie remained.
The next day, Cassie ate $\frac{2}{3}$ of the pie that remained.
This implies that $1 - \frac{2}{3} = \frac{1}{3}$ of the pie that was remaining was left after Cassie finished eating.
Thus, $\frac{1}{3}$ of $\frac{9}{20}$, or $\frac{3}{20}$ of the original pie was not eaten.

Answer: (D)

22. The first row is missing a 4 and a 2. Since there is already a 2 in the second column (in the second row), the first row, second column must contain a 4 and the first row, fourth column must contain a 2. To complete the upper left 2×2 square, the second row, first column must contain a 3. The second row is now missing both a 4 and a 1. But the fourth column already contains a 4 (in the fourth row), therefore the second row, fourth column must contain a 1. To complete the fourth column, we place a 3 in the third row. Now the P cannot be a 3, since there is already a 3 in the third row. Also, the P cannot be a 4 or a 2, since the second column already contains these numbers. By process of elimination, the digit 1 must replace the P.

Answer: (A)

23. *Solution 1*
We can suppose that the jug contains 1 litre of water at the start. The following table shows the quantity of water poured in each glass and the quantity of water remaining in each glass after each pouring, stopping when the quantity of water remaining is less than 0.5 L.

Number of glasses	Number of litres poured	Number of litres remaining
1	10 % of 1 = 0.1	$1 - 0.1 = 0.9$
2	10 % of 0.9 = 0.09	$0.9 - 0.09 = 0.81$
3	10 % of 0.81 = 0.081	$0.81 - 0.081 = 0.729$
4	10 % of 0.729 = 0.0729	$0.729 - 0.0729 = 0.6561$
5	10 % of 0.6561 = 0.06561	$0.6561 - 0.06561 = 0.59049$
6	10 % of 0.59049 = 0.059049	$0.59049 - 0.059049 = 0.531441$
7	10 % of 0.531441 = 0.0531441	$0.531441 - 0.0531441 = 0.4782969$

We can see from the table that the minimum number of glasses that Kim must pour so that less than half of the water remains in the jug is 7.

Solution 2
Removing 10% of the water from the jug is equivalent to leaving 90% of the water in the jug.

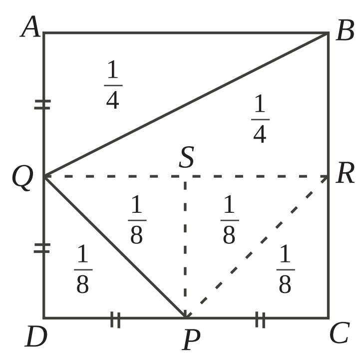
Thus, to find the total fraction remaining in the jug after a given pour, we multiply the previous total by 0.9.
We make the following table; stopping when the fraction of water remaining in the glass is first less than 0.5 (one half).

Number of glasses poured	Fraction of water remaining
1	$0.9 \times 1 = 0.9$
2	$0.9 \times 0.9 = 0.81$
3	$0.9 \times 0.81 = 0.729$
4	$0.9 \times 0.729 = 0.6561$
5	$0.9 \times 0.6561 = 0.59049$
6	$0.9 \times 0.59049 = 0.531441$
7	$0.9 \times 0.531441 = 0.4782969$

We can see from the table that the minimum number of glasses that Kim must pour so that less than half of the water remains in the jug is 7.

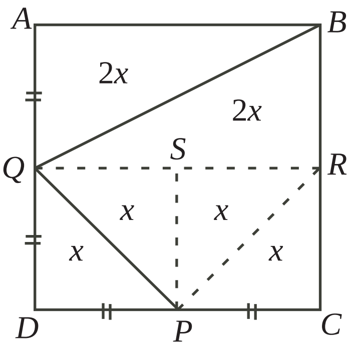
Answer: (C)

24. *Solution 1*
Draw line segment QR parallel to DC, as in the following diagram. This segment divides square $ABCD$ into two halves. Since triangles ABQ and RQB are congruent, each is half of rectangle $ABRQ$ and therefore one quarter of square $ABCD$. Draw line segment PS parallel to DA, and draw line segment PR. Triangles PDQ, PSQ, PSR and PCR are congruent. Therefore each is one quarter of rectangle $DCRQ$ and therefore one eighth of square $ABCD$.



Quadrilateral $QBCP$ therefore represents $\frac{1}{4} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{5}{8}$ of square $ABCD$. Its area is therefore $\frac{5}{8}$ of the area of the square.
Therefore, $\frac{5}{8}$ of the area of the square is equal to 15. Therefore, $\frac{1}{8}$ of the area of the square is equal to 3. Therefore the square has an area of 24.

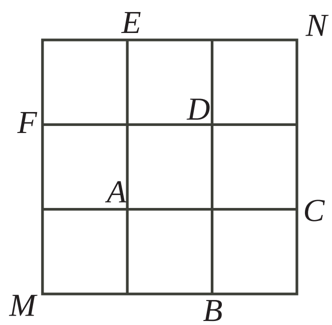
Solution 2
Draw a line segment from Q to R, the midpoint of BC.
Draw a line segment from P to S, the midpoint of QR.
Let the area of $\triangle QSP$ equal x. Thus, the area of $\triangle QDP$ is also x and $QDPS$ has area $2x$.
Square $SPCR$ is congruent to square $QDPS$ and thus has area $2x$.
Rectangle $QDCR$ has area $4x$, as does the congruent rectangle $AQRB$.
Also, $\triangle AQB$ and $\triangle BRQ$ have equal areas and thus, each area is $2x$.



Quadrilateral $QBCP$ is made up of $\triangle BRQ$, $\triangle QSP$ and square $SPCR$, and thus has area $2x + x + 2x = 5x$.
Since quadrilateral $QBCP$ has area 15, then $5x = 15$ or $x = 3$.
Therefore, the area of square $ABCD$, which is made up of quadrilateral $QBCP$, $\triangle AQB$ and $\triangle QDP$, is $5x + 2x + x = 8x = 8(3) = 24$.

ANSWER: (E)

25. Labeling the diagram as shown below, we can describe paths using the points they pass through.



The path $MADN$ is the only path of length 3 (traveling along 3 diagonals).
Since the diagram is symmetrical about MN, all other paths will have a *reflected path* in the line MN and therefore occur in pairs. This observation alone allows us to eliminate (B) and (D) as possible answers since they are even.
The following table lists all possible paths from M to N traveling along diagonals only.

Path length	Path Name	Reflected Path
3	$MADN$	same
5	$MABCDN$	$MAFEDN$
9	$MABCDEFADN$	$MAFEDCBADN$
	$MABCDAFEDN$	$MAFEDABCDN$
	$MADCBAFEDN$	$MADEFABCDN$

At this point we have listed 9 valid paths. Since paths occur in pairs (with the exception of $MADN$), the next possible answer would be 11. Since 11 is not given as an answer (and 9 is the largest possible answer given), we can be certain that we have found them all.

ANSWER: (E)

Grade 8

1. Using the correct order of operations, $1 + 3^2 = 1 + 9 = 10$.

   Answer: (B)

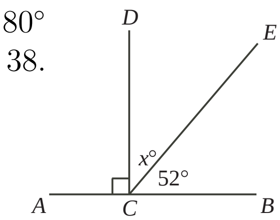
2. Calculating, $-10 + (-12) = -22$.

   Answer: (D)

3. Using a 1 litre jug of water, Jack could fill two 0.5 (one half) litre bottles of water.
   Using a 3 litre jug of water, Jack could fill $3 \times 2 = 6$ bottles of water.
   (Check: $3 \div 0.5 = 3 \div \frac{1}{2} = 3 \times 2 = 6$)

   Answer: (C)

4. Since AB is a line segment, $\angle ACD + \angle DCE + \angle ECB = 180^\circ$ or $90^\circ + x^\circ + 52^\circ = 180^\circ$ or $x^\circ = 180^\circ - 90^\circ - 52^\circ$ or $x = 38$.



   Answer: (B)

5. Calculating, $\frac{7}{9} = 7 \div 9 = 0.7777\cdots = 0.\overline{7}$. Rounded to 2 decimal places, $\frac{7}{9}$ is 0.78.

   Answer: (C)

6. The graph shows that vehicle X uses the least amount of fuel for a given distance.
   Therefore, it is the most fuel efficient vehicle and will travel the farthest using 50 litres of fuel.

   Answer: (D)

7. Kayla spent $\frac{1}{4} \times \$100 = \$25$ on rides and $\frac{1}{10} \times \$100 = \$10$ on food.
   The total that she spent was \$35.

   Answer: (E)

8. The polyhedron has 6 faces and 8 vertices.
   The equation $F + V - E = 2$ becomes $6 + 8 - E = 2$ or $14 - E = 2$ or $E = 14 - 2 = 12$.
   Therefore, the polyhedron has 12 edges.

   Answer: (A)

9. Eliminating multiple occurrences of the same letter, the word 'PROBABILITY' uses 9 different letters of the alphabet, A, B, I, L, O, P, R, T, and Y.
   Since there are 26 letters in the alphabet, the probability that Jeff picks one of the 9 different letters in 'PROBABILITY' is $\frac{9}{26}$.

   Answer: (A)

10. *Solution 1*
    Since the two numbers differ by 2 but add to 20, the smaller number must be 1 less than half of 20 while the larger number is 1 greater than half of twenty.
    The smaller number is 9 and the larger is 11.

    Solution 2
    Since the numbers differ by 2, let the smaller number be represented by x and the larger number be represented by $x + 2$.
    Since their sum is 20, then $x + x + 2 = 20$ or $2x = 18$ or $x = 9$.
    The smaller number is 9 and the larger is 11.

    Answer: (A)

11. Since $\angle ABC = \angle ACB$, then $\triangle ABC$ is isosceles and $AB = AC$.
Given that $\triangle ABC$ has a perimeter of 32, $AB + AC + 12 = 32$ or $AB + AC = 20$.
But $AB = AC$, so $2AB = 20$ or $AB = 10$.

Answer: (C)

12. Substituting $C = 10$, the equation $F = \frac{9}{5}C + 32$ becomes $F = \frac{9}{5} \times 10 + 32 = 18 + 32 = 50$.
A temperature of 10 degrees Celsius is equal to 50 degrees Fahrenheit.

Answer: (D)

13. Beginning with the positive integer 1 as a number in the first pair, we get the sum $101 = 1+100$.
From this point we can continue to increase the first number by one while decreasing the second number by one, keeping the sum equal to 101.
The list of possible sums is:

$$\begin{aligned} 101 &= 1 + 100 \\ 101 &= 2 + 99 \\ 101 &= 3 + 98 \\ &\vdots \\ 101 &= 50 + 51 \end{aligned}$$

After this point, the first number will no longer be smaller than the second if we continue to add 1 to the first number and subtract 1 from the second.
There are 50 possible sums in all.

Answer: (A)

14. The six other players on the team averaged 3.5 points each.
The total of their points was $6 \times 3.5 = 21$.
Vanessa scored the remainder of the points, or $48 - 21 = 27$ points.

Answer: (E)

15. Triangle PQR is a right-angled triangle since $\angle PQR = 90^\circ$ (because $PQRS$ is a rectangle).
In $\triangle PQR$, the Pythagorean Theorem gives,

$$\begin{aligned} PR^2 &= PQ^2 + QR^2 \\ 13^2 &= 12^2 + QR^2 \\ 169 &= 144 + QR^2 \\ 169 - 144 &= QR^2 \\ QR^2 &= 25 \end{aligned}$$

So $QR = 5$ since $QR > 0$. The area of $PQRS$ is thus $12 \times 5 = 60$.

Answer: (B)

16. When it is 3:00 p.m. in Victoria, it is 6:00 p.m. in Timmins.
Therefore, Victoria time is always 3 hours earlier than Timmins time.
When the flight arrived at 4:00 p.m. local Timmins time, the time in Victoria was 1:00 p.m.
The plane left at 6:00 a.m. Victoria time and arrived at 1:00 p.m. Victoria time.
The flight was 7 hours long.

Answer: (D)

17. The value of all quarters is \$10.00.
Each quarter has a value of \$0.25.
There are thus $10 \div 0.25 = 40$ quarters in the jar.
Similarly, there are $10 \div 0.05 = 200$ nickels, and $10 \div 0.01 = 1000$ pennies in the jar.
In total, there are $40 + 200 + 1000 = 1240$ coins in the jar.
The probability that the selected coin is a quarter is $\dfrac{\text{the number of quarters}}{\text{the total number of coins}} = \dfrac{40}{1240} = \dfrac{1}{31}$.

Answer: (B)

18. Of the 40 students, 12 did not like either dessert.
Therefore, $40 - 12 = 28$ students liked at least one of the desserts.
But 18 students said they liked apple pie, 15 said they liked chocolate cake, and $18 + 15 = 33$, so $33 - 28 = 5$ students must have liked both of the desserts.

Answer: (E)

19. In the ones column, $P + Q$ ends in 9. So $P + Q = 9$ or $P + Q = 19$.
Since P is at most 9 and Q is at most 9, then $P + Q$ is at most 18.
Therefore, $P + Q = 9$ since $P + Q$ cannot equal 19.
In the tens column, since $Q + Q$ ends in zero, $Q + Q$ equals 0 or 10.
Therefore, either $Q = 0$ or $Q = 5$.
If $Q = 0$, there would be no "carry" to the hundreds column where $P + Q$ (plus no carry) ends in a zero.
This is not possible since we already determined that $P + Q = 9$.
Therefore, $Q = 5$ and $P = 4$, giving a 1 carried from the tens column to the hundreds column.
In the hundreds column, we have $1 + 4 + 5$ which gives a 1 carried from the hundreds column to the thousands column.
Then R plus the 1 carried equals 2, so $R = 1$. Thus, $P + Q + R = 4 + 5 + 1 = 10$.

$$\begin{array}{rcccc} & & P & Q & P \\ + & R & Q & Q & Q \\ \hline & 2 & 0 & 0 & 9 \end{array} \qquad \begin{array}{rcccc} & & 4 & 5 & 4 \\ + & 1 & 5 & 5 & 5 \\ \hline & 2 & 0 & 0 & 9 \end{array}$$

Answer: (B)

20. Since the area of the square is 144, each side has length $\sqrt{144} = 12$.
The length of the string equals the perimeter of the square which is $4 \times 12 = 48$.
The largest circle that can be formed from this string has a circumference of 48 or $2\pi r = 48$.
Solving for the radius r, we get $r = \frac{48}{2\pi} \approx 7.64$.
The maximum area of a circle that can be formed using the string is $\pi(\frac{48}{2\pi})^2 \approx \pi(7.64)^2 \approx 183$.

Answer: (E)

21. For the sum to be a maximum, we must choose the three smallest divisors in an effort to make the fourth divisor as large as possible.
The smallest 3 divisors of 360 are 1, 2 and 3, making $\frac{360}{1\times2\times3} = 60$ the fourth divisor.
We note here that 1, 2 and 3 are the smallest three different divisors of 360.
Therefore, it is not possible to use a divisor greater than 60, since there aren't three divisors smaller than 1, 2 and 3.
Replacing the divisor 60 with a smaller divisor will decrease the sum of the four divisors.
To see this, we recognize that the product of 3 different positive integers is always greater than

or equal to the sum of the 3 integers. For example, $1 \times 2 \times 4 = 8 > 1 + 2 + 4 = 7$.
The next largest divisor less than 60 is 45, thus the remaining three divisors would have a product of $360 \div 45 = 8$, and therefore have a sum that is less than or equal to 8.
This gives a combined sum that is less than or equal to $45+8 = 53$, much less than the previous sum of $1 + 2 + 3 + 60 = 66$. In the same way, we obtain sums smaller than 66 if we consider the other divisors of 360 as the largest of the four integers. Therefore, the maximum possible sum is 66.

Answer: (B)

22. The first vertical line through the letter S cuts the S into 4 pieces, 2 pieces to the left of the line and 2 to the right.
Each additional vertical line adds 3 new pieces while preserving the 4 original pieces.
The chart below shows the number of pieces increasing by three with each additional line drawn.

Lines	Pieces
1	4
2	7
3	10
4	13
⋮	⋮

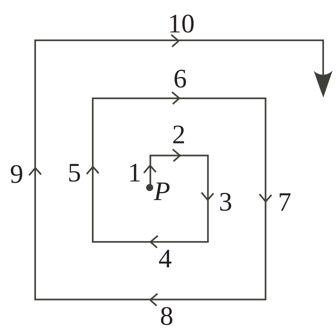
We want 154 pieces. Since the first line gives 4 pieces, we require an additional 150 pieces. Since three new pieces are created for each additional line drawn, we need to add $150 \div 3 = 50$ new lines after the first, or 51 lines in total.

Answer: (D)

23. Since we are looking for the percentage of the whole length, we may take the side length of the square to be any convenient value, as the actual length will not affect the final answer.
Let us assume that the side length of the square is 2. Then the diameter of the circle is also 2 because the width of the square and the diameter of the circle are equal.
Using the Pythagorean Theorem, $XY^2 = 2^2 + 2^2 = 4 + 4 = 8$ or $XY = \sqrt{8}$.
The portion of line segment XY lying outside the circle has length XY minus the diameter of the circle, or $\sqrt{8} - 2$.
The percentage of line segment XY lying outside the circle is $\dfrac{\sqrt{8} - 2}{\sqrt{8}} \times 100\% \approx 29.3\%$.

Answer: (A)

24. *Solution 1*
Breenah travels along each of the sides in a direction that is either up, down, right or left.
The "up" sides occur every fourth segment, thus they have lengths $1, 5, 9, 13, 17, 21, \ldots$ or lengths that are one more than a multiple of 4. As we see, the segment of length 21 is an up side.

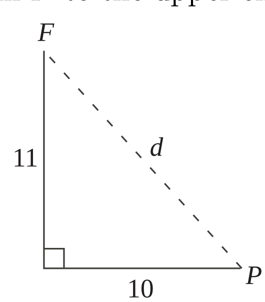


We see that the upper endpoint of each up segment is 2 units to the left and 2 units above the upper endpoint of the previous up segment. Thus, when Breenah is standing at the upper endpoint of the up segment of length 21, she is $2+2+2+2+2=10$ units to the left and $1+2+2+2+2+2=11$ units above P. The following paragraphs prove this in a more formal way.

We now determine the horizontal distance from point P to the up side of length 21.
Following the spiral outward from P, the first horizontal line segment moves right 2, or $+2$, where the positive sign indicates movement to the right.
The second horizontal segment moves left 4, or -4, where the negative sign indicates movement to the left.
After these two horizontal movements, we are at the line segment of length 5, an up side. To get there, we moved a horizontal distance of $(+2)+(-4)=-2$ or 2 units to the left.
The horizontal distance from P to the next up side (length 9), can be found similarly.
Beginning on the segment of length 5, we are already at -2 or 2 units left of P and we move right 6 (or $+6$), then left 8 (or -8).
Thus, to reach the up side with length 9, we have moved horizontally
$(-2)+(+6)+(-8)=-4$ or 4 units left of P.
This pattern of determining the horizontal distances from P to each of the up sides is continued in the table below.
We now determine the vertical distance from point P to the upper endpoint of the up side with length 21.
From P, the first vertical segment moves up 1 or $+1$, the second moves down 3 or -3 and the third moves up 5 or $+5$.
Therefore the vertical position of the endpoint of the up side with length 5 is
$(+1)+(-3)+(+5)=+3$ or 3 units above P.
Similarly, we can calculate the vertical position of each of the up side endpoints relative to P and have summarized this in the table below.

Side Length	Horizontal Distance	Vertical Distance
5	$(+2)+(-4)=-2$	$(+1)+(-3)+(+5)=+3$
9	$(-2)+(+6)+(-8)=-4$	$(+3)+(-7)+(+9)=+5$
13	$(-4)+(+10)+(-12)=-6$	$(+5)+(-11)+(+13)=+7$
17	$(-6)+(+14)+(-16)=-8$	$(+7)+(-15)+(+17)=+9$
21	$(-8)+(+18)+(-20)=-10$	$(+9)+(-19)+(+21)=+11$

We now calculate the distance, d, from P to the upper endpoint, F, of the up side of length 21.



Since the calculated distances are horizontal and vertical, we have created a right angle and may find the required distance using the Pythagorean Theorem.
Then $d^2=10^2+11^2=100+121=221$ or $d=\sqrt{221}\approx 14.866$.

Solution 2

If we place the spiral on an xy-plane with point P at the origin, the coordinates of the key points reveal a pattern.

Side Length	Endpoint Coordinates
1	$(0, 1)$
2	$(2, 1)$
3	$(2, -2)$
4	$(-2, -2)$
5	$(-2, 3)$
6	$(4, 3)$
7	$(4, -4)$
8	$(-4, -4)$
$\vdots$	$\vdots$

From the table we can see that after finishing a side having length that is a multiple of 4, say $4k$, we are at the point $(-2k, -2k)$ (the basis for this argument is shown in solution 1).
Therefore, after completing the side of length 20, we are at the point $(-10, -10)$.
All sides of length $4k$ travel toward the left.
We must now move vertically upward 21 units from this point $(-10, -10)$.
Moving upward, this last side of length 21 will end at the point $F(-10, 11)$.
This point is left 10 units and up 11 units from $P(0, 0)$.
Using the Pythagorean Theorem, $PF^2 = 10^2 + 11^2 = 100 + 121 = 221$ or $PF = \sqrt{221} \approx 14.866$.

ANSWER: (B)

25. Consider all possible sums of pairs of numbers that include p: $p + q$, $p + r$, $p + s$, $p + t$ and $p + u$.
We see that p is included in 5 different sums (once with each of the other 5 numbers in the list).
Similarly, each of the numbers will be included in 5 sums.
If the sums of all 15 pairs are added together, each of p, q, r, s, t, and u will have been included 5 times.
Therefore, $5p + 5q + 5r + 5s + 5t + 5u = 25 + 30 + 38 + 41 + \ldots + 103 + 117 = 980$ or $5(p + q + r + s + t + u) = 980$.
Dividing this equation by 5, we obtain $p + q + r + s + t + u = \frac{980}{5} = 196$.
Since p and q are the smallest two integers, their sum must be the smallest of all of the pairs; thus, $p + q = 25$.
Similarly, t and u are the largest two integers and their sum must be the largest of all of the pairs; thus, $t + u = 117$.
Now, $(p + q) + r + s + (t + u) = 196$ becomes $25 + r + s + 117 = 196$ or $r + s = 196 - 142 = 54$.

ANSWER: (B)