2009 Fryer Contest (Grade 9)

Wednesday, April 8, 2009

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1. Emily sets up a lemonade stand. She has set-up costs of \$12.00 and each cup of lemonade costs her \$0.15 to make. She sells each cup of lemonade for \$0.75.

   (a) What is the total cost, including the set-up, for her to make 100 cups of lemonade?

   (b) What is her profit (money earned minus total cost) if she sells 100 cups of lemonade?

   (c) What is the number of cups that she must sell to break even (that is, to have a profit of \$0)?

   (d) Why is it not possible for her to make a profit of exactly \$17.00?

2. If $a > 0$ and $b > 0$, a new operation ∇ is defined as follows: $a \nabla b = \dfrac{a+b}{1+ab}$.

   For example, $3 \nabla 6 = \dfrac{3+6}{1+3\times 6} = \dfrac{9}{19}$.

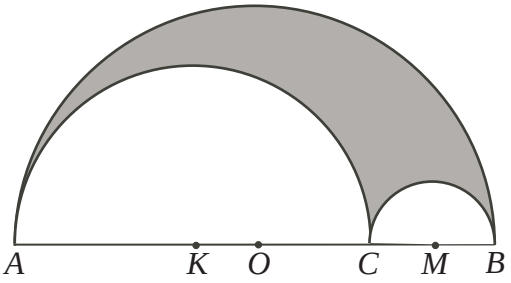
   (a) Calculate $2 \nabla 5$.

   (b) Calculate $(1 \nabla 2) \nabla 3$.

   (c) If $2 \nabla x = \dfrac{5}{7}$, what is the value of x?

   (d) For some values of x and y, the value of $x \nabla y$ is equal to $\dfrac{x+y}{17}$. Determine all possible ordered pairs of positive integers x and y for which this is true.

3. In the diagram, K, O and M are the centres of the three semi-circles. Also, $OC = 32$ and $CB = 36$.

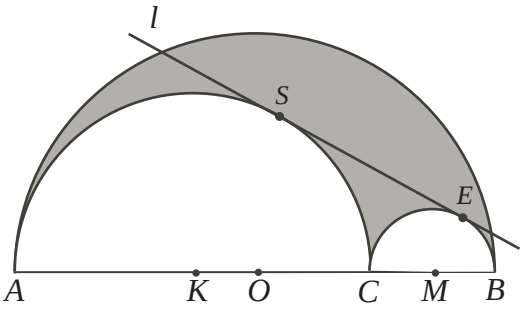


   (a) What is the length of AC?

   (b) What is the area of the semi-circle with centre K?

   (c) What is the area of the shaded region?

   (d) Line l is drawn to touch the smaller semi-circles at points S and E so that KS and ME are both perpendicular to l. Determine the area of quadrilateral $KSEM$.



4. The addition shown below, representing $2 + 22 + 222 + 2222 + \cdots$, has 101 rows and the last term consists of 101 2's:

$$\begin{array}{rcccccccc}
 & & & & & & & & 2 \\
 & & & & & & & 2 & 2 \\
 & & & & & & 2 & 2 & 2 \\
 & & & & & 2 & 2 & 2 & 2 \\
 & & & & & & & & \vdots \\
 & & & 2 & 2 & \cdots & 2 & 2 & 2 & 2 \\
+ & 2 & 2 & 2 & 2 & \cdots & 2 & 2 & 2 & 2 \\
\hline
 & & & & & \cdots & C & B & A
\end{array}$$

(a) Determine the value of the ones digit A .

(b) Determine the value of the tens digit B and the value of the hundreds digit C.

(c) Determine the middle digit of the sum.