2008 Hypatia Contest (Grade 11)

Wednesday, April 16, 2008

---

1. For numbers a and b, the notation $a\nabla b$ means $2a + b^2 + ab$.
   For example, $1\nabla 2 = 2(1) + 2^2 + (1)(2) = 8$.

   (a) Determine the value of $3\nabla 2$.

   (b) If $x\nabla(-1) = 8$, determine the value of x.

   (c) If $4\nabla y = 20$, determine the two possible values of y.

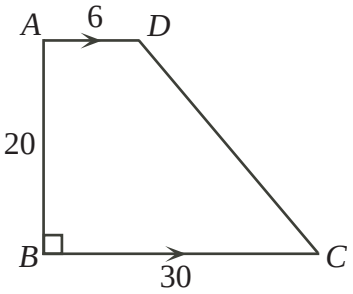
   (d) If $(w-2)\nabla w = 14$, determine all possible values of w.

2. (a) Determine the equation of the line through the points $A(7, 8)$ and $B(9, 0)$.

   (b) Determine the coordinates of P, the point of intersection of the line $y = 2x - 10$ and the line through A and B.

   (c) Is P closer to A or to B? Explain how you obtained your answer.

3. In the diagram, $ABCD$ is a trapezoid with AD parallel to BC and BC perpendicular to AB. Also, $AD = 6$, $AB = 20$, and $BC = 30$.



   (a) Determine the area of trapezoid $ABCD$.

   (b) There is a point K on AB such that the area of $\triangle KBC$ equals the area of quadrilateral $KADC$. Determine the length of BK.

   (c) There is a point M on DC such that the area of $\triangle MBC$ equals the area of quadrilateral $MBAD$. Determine the length of MC.

4. The *peizi-sum* of a sequence $a_1, a_2, a_3, \ldots, a_n$ is formed by adding the products of all of the pairs of distinct terms in the sequence. For example, the peizi-sum of the sequence a_1, a_2, a_3, a_4 is $a_1a_2 + a_1a_3 + a_1a_4 + a_2a_3 + a_2a_4 + a_3a_4$.

   (a) The peizi-sum of the sequence $2, 3, x, 2x$ is -7. Determine the possible values of x.

   (b) A sequence has 100 terms. Of these terms, m are equal to 1 and n are equal to -1. The rest of the terms are equal to 2. Determine, in terms of m and n, the number of pairs of distinct terms that have a product of 1.

   (c) A sequence has 100 terms, with each term equal to either 2 or -1. Determine, with justification, the minimum possible peizi-sum of the sequence.