Canadian Mathematics Competition

An activity of the Centre for Education in Mathematics and Computing, University of Waterloo, Waterloo, Ontario

Grade 8 solutions follow the Grade 7 solutions

2007 Gauss Contests

(Grades 7 and 8)

Wednesday, May 16, 2007

Solutions

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Grade 7

Grade 8 solutions follow the Grade 7 solutions

1. Calculating, $(4-3)\times 2 = 1\times 2 = 2$.

   Answer: (B)

2. Since one thousand is 1000, then ten thousand is 10 000.

   Answer: (C)

3. When we subtract 5 from the missing number, the answer is 2, so to find the missing number, we add 5 to 2 and obtain 7. (Check: $7-5=2$.)

   Answer: (A)

4. *Solution 1*
   As a fraction, 80% is $\frac{80}{100}$ or $\frac{4}{5}$.
   Therefore, Mukesh got $\frac{4}{5}$ of the possible 50 marks, or $\frac{4}{5}\times 50 = 40$ marks.

   Solution 2
   Since Mukesh got 80% of the 50 marks, he got $\frac{80}{100}\times 50 = \frac{80}{2} = 40$ marks.

   Answer: (A)

5. *Solution 1*

$$\begin{aligned}\frac{7}{10}+\frac{3}{100}+\frac{9}{1000} &= \frac{700}{1000}+\frac{30}{1000}+\frac{9}{1000} \qquad \text{(using a common denominator)}\\ &= \frac{739}{1000}\\ &= 0.739\end{aligned}$$

   Solution 2

$$\begin{aligned}\frac{7}{10}+\frac{3}{100}+\frac{9}{1000} &= 0.7+0.03+0.009 \qquad \text{(converting each fraction to a decimal)}\\ &= 0.739\end{aligned}$$

   Answer: (D)

6. *Solution 1*
   Mark has $\frac{3}{4}$ of a dollar, or 75 cents.
   Carolyn has $\frac{3}{10}$ of a dollar, or 30 cents.
   Together, they have $75+30=105$ cents, or \$1.05.

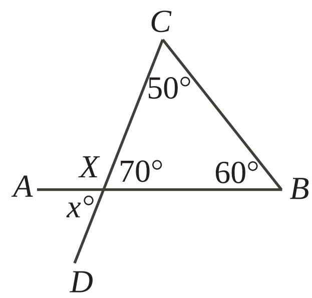
   Solution 2
   Since Mark has $\frac{3}{4}$ of a dollar and Carolyn has $\frac{3}{10}$ of a dollar, then together they have $\frac{3}{4}+\frac{3}{10}=\frac{15}{20}+\frac{6}{20}=\frac{21}{20}$ of a dollar.
   Since $\frac{21}{20}$ is equivalent to $\frac{105}{100}$, they have \$1.05.

   Answer: (E)

7. From the graph, the student who ate the most apples ate 6 apples, so Lorenzo ate 6 apples. Also from the graph, the student who ate the fewest apples ate 1 apple, so Jo ate 1 apple. Therefore, Lorenzo ate $6-1=5$ more apples than Jo.

   Answer: (B)

8. Since the angles in a triangle add to 180°, then the missing angle in the triangle is $180^\circ - 50^\circ - 60^\circ = 70^\circ$.
We then have:



Since $\angle BXC = 70^\circ$, then $\angle AXC = 180^\circ - \angle BXC = 110^\circ$.
Since $\angle AXC = 110^\circ$, then $\angle DXA = 180^\circ - \angle AXC = 70^\circ$.
Therefore, $x = 70$.
(Alternatively, we could note that when two lines intersect, the vertically opposite angles are equal so $\angle DXA = \angle BXC = 70^\circ$.)

ANSWER: (E)

9. When the word BANK is viewed from the inside of the window, the letters appear in the reverse order and the letters themselves are all backwards, so the word appears as ꓘИAᗺ.

ANSWER: (D)

10. Since a large box costs \$3 more than a small box and a large box and a small box together cost \$15, then replacing the large box with a small box would save \$3.
This tells us that two small boxes together cost \$12.
Therefore, one small box costs \$6.

ANSWER: (D)

11. Since each number in the Fibonacci sequence, beginning with the 2, is the sum of the two previous numbers, then the sequence continues as $1, 1, 2, 3, 5, 8, 13, 21$.
Thus, 21 appears in the sequence.

ANSWER: (B)

12. The probability that Mary wins the lottery is equal to the number of tickets that Mary bought divided by the total number of tickets in the lottery.
We are told that the probability that Mary wins is $\frac{1}{15}$.
Since there were 120 tickets in total sold, we would like to write $\frac{1}{15}$ as a fraction with 120 in the denominator.
Since $120 \div 15 = 8$, then we need to multiply the numerator and denominator of $\frac{1}{15}$ each by 8 to obtain a denominator of 120.
Therefore, the probability that Mary wins is $\dfrac{1 \times 8}{15 \times 8} = \dfrac{8}{120}$. Since there were 120 tickets sold, then Mary must have bought 8 tickets.

ANSWER: (D)

13. *Solution 1*
We look at each of the choices and try to make them using only 3 cent and 5 cent stamps:

(A): 7 cannot be made, since no more than one 5 cent and two 3 cent stamps could be used (try playing with the possibilities!)
(B): $13 = 5 + 5 + 3$
(C): 4 cannot be the answer since a larger number (7) already cannot be made
(D): $8 = 5 + 3$
(E): $9 = 3 + 3 + 3$

Therefore, the answer must be 7.
(We have not really justified that 7 is the largest number that cannot be made using only 3s and 5s; we have, though, determined that 7 must be the answer to this question, since it is the only possible answer from the given possibilities! See Solution 2 for a justification that 7 is indeed the answer.)

Solution 2
We make a table to determine which small positive integers can be made using 3s and 5s:

Integer	Combination
1	Cannot be made
2	Cannot be made
3	3
4	Cannot be made
5	5
6	$3 + 3$
7	Cannot be made
8	$5 + 3$
9	$3 + 3 + 3$
10	$5 + 5$
11	$5 + 3 + 3$

Every integer larger than 11 can also be made because the last three integers in our table can be made and we can add a 3 to our combinations for 9, 10 and 11 to get combinations for 12, 13 and 14, and so on.
From the table, the largest amount of postage that cannot be made is 7.

ANSWER: (A)

14. We list all of the possible orders of finish, using H, R and N to stand for Harry, Ron and Neville.
The possible orders are HNR, HRN, NHR, NRH, RHN, RNH.
(It is easiest to list the orders in alphabetical order to better keep track of them.)
There are 6 possible orders.

ANSWER: (B)

15. *Solution 1*
The positive whole numbers that divide exactly into 40 are 1, 2, 4, 5, 8, 10, 20, 40.
The positive whole numbers that divide exactly into 72 are 1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72.
The numbers that occur in both lists are 1, 2, 4, 8, or four numbers in total.

Solution 2
The greatest common divisor of 40 and 72 is 8.
Any common divisor of 40 and 72 is a divisor of the greatest common divisor (namely 8) and vice-versa.
Since the positive divisors of 8 are 1, 2, 4, and 8, there are four such common positive divisors.

Answer: (C)

16. The first scale tells us that a square and a circle together have a mass of 8.
The second scale tells us that a square and two circles together have a mass of 11.
We can replace the square and one circle on the second scale with an "8", so 8 plus the mass of a circle gives a mass of 11. This tells us that the mass of a circle is 3.
From the third scale, since the mass of a circle and two triangles is 15, then the mass of the two triangles only is $15 - 3 = 12$.
Therefore, the mass of one triangle is $12 \div 2 = 6$.

Answer: (D)

17. The total cost to use the kayak for 3 hours is $3 \times \$5 = \$15$. Since the total rental cost for 3 hours is \$30, then the fixed fee to use the paddle is $\$30 - \$15 = \$15$.
For a six hour rental, the total cost is thus $\$15 + (6 \times \$5) = \$15 + \$30 = \$45$.

Answer: (C)

18. *Solution 1*
Julie's birthday was $37 + 67 = 104$ days before Fred's birthday.
When we divide 104 by 7 (the number of days in one week), we obtain a quotient of 14 and a remainder of 6.
In 14 weeks, there are $14 \times 7 = 98$ days, so 98 days before Fred's birthday was also a Monday.
Since Julie's birthday was 104 days before Fred's, this was 6 days still before the Monday 98 days before Fred's birthday. The 6th day before a Monday is a Tuesday.
Therefore, Julie's birthday was a Tuesday.

Solution 2
37 days is 5 weeks plus 2 days. Since Fred's birthday was on a Monday and Pat's birthday was 37 days before Fred's, then Pat's birthday was on a Saturday.
67 days is 9 weeks plus 4 days. Since Pat's birthday was on a Saturday and Julie's birthday was 67 days before Pat's, then Julie's birthday was on a Tuesday.

Answer: (D)

19. The positive whole numbers less than 1000 that end with 77 are 77, 177, 277, 377, 477, 577, 677, 777, 877, 977.
The positive whole numbers less than 1000 which begin with 77 are 77, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779.
There is no other way for a positive whole number less than 1000 to contain at least two 7's side-by-side.
There are 10 numbers in the first list and 11 numbers in the second list. Since 2 numbers appear in both lists, the total number of whole numbers in the two lists is $10 + 11 - 2 = 19$.

Answer: (E)

20. Since the perimeter of the square is 48, its side length is $48 \div 4 = 12$.
Since the side length of the square is 12, its area is $12 \times 12 = 144$.
The area of the triangle is $\frac{1}{2} \times 48 \times x = 24x$.
Since the area of the triangle equals the area of the square, then $24x = 144$ or $x = 6$.

ANSWER: (C)

21. *Solution 1*
Starting at the "K" there are two possible paths that can be taken. At each "A", there are again two possible paths that can be taken. Similarly, at each "R" there are two possible paths that can be taken.
Therefore, the total number of paths is $2 \times 2 \times 2 = 8$.
(We can check this by actually tracing out the paths.)

Solution 2
Each path from the K at the top to one of the L's at the bottom has to spell KARL.
There is 1 path that ends at the first L from the left. This path passes through the first A and the first R.
There are 3 paths that end at the second L. The first of these passes through the first A and the first R. The second of these passes through the first A and the second R. The third of these passes through the second A and the second R.
There are 3 paths that end at the third L. The first of these passes through the first A and the second R. The second of these passes through the second A and the second R. The third of these passes through the second A and the third R.
There is 1 path that ends at the last L. This path passes through the last A and the last R.
So the total number of paths to get to the bottom row is $1+3+3+1 = 8$, which is the number of paths that can spell KARL.

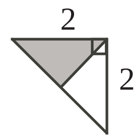
ANSWER: (D)

22. Since the average of four numbers is 4, their sum is $4 \times 4 = 16$.
For the difference between the largest and smallest of these numbers to be as large as possible, we would like one of the numbers to be as small as possible (so equal to 1) and the other (call it B for big) to be as large as possible.
Since one of the numbers is 1, the sum of the other three numbers is $16 - 1 = 15$.
For the B to be as large as possible, we must make the remaining two numbers (which must be different and not equal to 1) as small as possible. So these other two numbers must be equal to 2 and 3, which would make B equal to $15 - 2 - 3 = 10$.
So the average of these other two numbers is $\dfrac{2+3}{2} = \dfrac{5}{2}$ or $2\frac{1}{2}$.

ANSWER: (B)

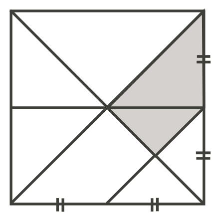
23. *Solution 1*
Since we are dealing with fractions of the whole area, we may make the side of the square any convenient value.
Let us assume that the side length of the square is 4.
Therefore, the area of the whole square is $4 \times 4 = 16$.
The two diagonals of the square divide it into four pieces of equal area (so each piece has area $16 \div 4 = 4$).
The shaded area is made up from the "right" quarter of the square with a small triangle removed, and so has area equal to 4 minus the area of this small triangle.

This small triangle is half of a larger triangle.



This larger triangle has its base and height each equal to half of the side length of the square (so equal to 2) and has a right angle. So the area of this larger triangle is $\frac{1}{2} \times 2 \times 2 = 2$.
So the area of the small triangle is $\frac{1}{2} \times 2 = 1$, and so the area of the shaded region is $4 - 1 = 3$.
Therefore, the shaded area is $\frac{3}{16}$ of the area of the whole square.

Solution 2
Draw a horizontal line from the centre of the square through the shaded region.



The two diagonals divide the square into four pieces of equal area. The new horizontal line divides one of these pieces into two parts of equal area. Therefore, the shaded region above the new horizontal line is $\frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$ of the total area of the square.
The shaded piece below this new horizontal line is half of the bottom right part of this right-hand piece of the square. (It is half of this part because the shaded triangle and unshaded triangle making up this part have the same shape.) So this remaining shaded piece is $\frac{1}{2} \times \frac{1}{8} = \frac{1}{16}$ of the total area of the square.
In total, the shaded region is $\frac{1}{8} + \frac{1}{16} = \frac{3}{16}$ of the total area of the square.

Answer: (C)

24. First, we try to figure out what digit Q is.
Since the product is not equal to 0, Q cannot be 0. Since the product has four digits and the top number has three digits, then Q (which is multiplying the top number) must be bigger than 1.
Looking at the units digits in the product, we see that $Q \times Q$ has a units digit of Q.
Since $Q > 1$, then Q must equal 5 or 6 (no other digit gives itself as a units digit when multiplied by itself).
But Q cannot be equal to 5, since if it was, the product $RQ5Q$ would end "55" and each of the two parts (PPQ and Q) of the product would end with a 5. This would mean that each of the parts of the product was divisible by 5, so the product should be divisible by $5 \times 5 = 25$. But a number ending in 55 is not divisible by 25.
Therefore, $Q = 6$.
So the product now looks like

$$\begin{array}{r} PP6 \\ \times \quad 6 \\ \hline R656 \end{array}$$

Now when we start the long multiplication, 6×6 gives 36, so we write down 6 and carry a 3.
When we multiply $P \times 6$ and add the carry of 3, we get a units digit of 5, so the units digit of

$P \times 6$ should be 2.
For this to be the case, $P = 2$ or $P = 7$.
We can now try these possibilities: $226 \times 6 = 1356$ and $776 \times 6 = 4656$. Only the second ends "656" like the product should.
So $P = 7$ and $R = 4$, and so $P + Q + R = 7 + 6 + 4 = 17$.

ANSWER: (E)

25. The easiest way to keep track of the letters here is to make a table of what letters arrive at each time, what letters are removed, and what letters stay in the pile.

Time	Letters Arrived	Letters Removed	Remaining Pile (bottom to top)
12:00	1, 2, 3	3, 2	1
12:05	4, 5, 6	6, 5	1, 4
12:10	7, 8, 9	9, 8	1, 4, 7
12:15	10, 11, 12	12, 11	1, 4, 7, 10
12:20	13, 14, 15	15, 14	1, 4, 7, 10, 13
12:25	16, 17, 18	18, 17	1, 4, 7, 10, 13, 16
12:30	19, 20, 21	21, 20	1, 4, 7, 10, 13, 16, 19
12:35	22, 23, 24	24, 23	1, 4, 7, 10, 13, 16, 19, 22
12:40	25, 26, 27	27, 26	1, 4, 7, 10, 13, 16, 19, 22, 25
12:45	28, 29, 30	30, 29	1, 4, 7, 10, 13, 16, 19, 22, 25, 28
12:50	31, 32, 33	33, 32	1, 4, 7, 10, 13, 16, 19, 22, 25, 28, 31
12:55	34, 35, 36	36, 35	1, 4, 7, 10, 13, 16, 19, 22, 25, 28, 31, 34
1:00	None	34, 31	1, 4, 7, 10, 13, 16, 19, 22, 25, 28
1:05	None	28, 25	1, 4, 7, 10, 13, 16, 19, 22
1:10	None	22, 19	1, 4, 7, 10, 13, 16
1:15	None	16, 13	1, 4, 7, 10

(At 12:55, all 36 letters have been delivered, so starting at 1:00 letters are only removed and no longer added.)
Letter #13 is removed at 1:15.

ANSWER: (A)

Grade 8

1. Calculating, $(4 \times 12) - (4 + 12) = 48 - 16 = 32$.

   ANSWER: (E)

2. Converting to decimals, $\frac{3}{10} + \frac{3}{1000} = 0.3 + 0.003 = 0.303$.

   ANSWER: (B)

3. We use the graph to determine the difference between the high and low temperature each day.

Day	High	Low	Difference
Monday	20°	10°	10°
Tuesday	20°	15°	5°
Wednesday	25°	25°	0°
Thursday	30°	10°	20°
Friday	25°	20°	5°

   Therefore, the difference was the greatest on Thursday.

   ANSWER: (D)

4. When the cube is tossed, the total number of possibilities is 6 and the number of desired outcomes is 2.
   So the probability of tossing a 5 or 6 is $\frac{2}{6}$ or $\frac{1}{3}$.

   ANSWER: (C)

5. Since the side length of the cube is x cm, its volume is x^3 cm^3.
   Since the volume is known to be 8 cm^3, then $x^3 = 8$ so $x = 2$ (the cube root of 8).

   ANSWER: (A)

6. Since a 3 minute phone call costs \$0.18, then the rate is $\$0.18 \div 3 = \$0.06$ per minute.
   For a 10 minute call, the cost would be $10 \times \$0.06 = \$0.60$.

   ANSWER: (B)

7. Since there are 1000 metres in a kilometre, then 200 metres is equivalent to $\dfrac{200}{1000}$ km or 0.2 km.

   ANSWER: (A)

8. The children in the Gauss family have ages 7, 7, 7, 14, 15.
   The mean of their ages is thus $\dfrac{7+7+7+14+15}{5} = \dfrac{50}{5} = 10$.

   ANSWER: (E)

9. Since $x = 5$ and $y = x + 3$, then $y = 5 + 3 = 8$.
   Since $y = 8$ and $z = 3y + 1$, then $z = 3(8) + 1 = 24 + 1 = 25$.

   ANSWER: (B)

10. The possible three-digit numbers that can be formed using the digits 5, 1 and 9 are: 519, 591, 951, 915, 195, 159.
    The largest of these numbers is 951 and the smallest is 159.
    The difference between these numbers is $951 - 159 = 792$.

    ANSWER: (C)

11. *Solution 1*
Since Lily is 90 cm tall, Anika is $\frac{4}{3}$ as tall as Lily, and Sadaf is $\frac{5}{4}$ as tall as Anika, then Sadaf is $\frac{5}{4} \times \frac{4}{3} \times 90 = \frac{5}{3} \times 90 = \frac{450}{3} = 150$ cm tall.

Solution 2
Since Lily is 90 cm tall and Anika is $\frac{4}{3}$ of her height, then Anika is $\frac{4}{3} \times 90 = \frac{360}{3} = 120$ cm tall.
Since Anika is 120 cm tall and Sadaf is $\frac{5}{4}$ of her height, then Sadaf is $\frac{5}{4} \times 120 = \frac{600}{4} = 150$ cm tall.

Answer: (E)

12. *Solution 1*
Since $\angle BCA = 40^\circ$ and $\triangle ADC$ is isosceles with $AD = DC$, then $\angle DAC = \angle ACD = 40^\circ$.
Since the sum of the angles in a triangle is 180°, then
$\angle ADC = 180^\circ - \angle DAC - \angle ACD = 180^\circ - 40^\circ - 40^\circ = 100^\circ$.
Since $\angle ADB$ and $\angle ADC$ are supplementary, then $\angle ADB = 180^\circ - \angle ADC = 180^\circ - 100^\circ = 80^\circ$.
Since $\triangle ADB$ is isosceles with $AD = DB$, then $\angle BAD = \angle ABD$.
Thus, $\angle BAD = \frac{1}{2}(180^\circ - \angle ADB) = \frac{1}{2}(180^\circ - 80^\circ) = \frac{1}{2}(100^\circ) = 50^\circ$.
Therefore, $\angle BAC = \angle BAD + \angle DAC = 50^\circ + 40^\circ = 90^\circ$.

Solution 2
Since $\triangle ABD$ and $\triangle ACD$ are isosceles, then $\angle BAD = \angle ABD$ and $\angle DAC = \angle ACD$.
These four angles together make up all of the angles of $\triangle ABC$, so their sum is 180°.
Since $\angle BAC$ is half of the the sum of these angles (as it incorporates one angle of each pair), then $\angle BAC = \frac{1}{2}(180^\circ) = 90^\circ$.

Answer: (D)

13. For each of the 2 art choices, Cayli can chose 1 of 3 sports choices and 1 of 4 music choices. So for each of the 2 art choices, Cayli has $3 \times 4 = 12$ possible combinations of sports and music.
Since Cayli has 2 art choices, her total number of choices is $2 \times 12 = 24$.

Answer: (B)

14. At the 2007 Math Olympics, Canada won 17 of 100 possible medals, or 0.17 of the possible medals.
We convert each of the possible answers to a decimal and see which is closest to 0.17:

(A) $\frac{1}{4} = 0.25$ (B) $\frac{1}{5} = 0.2$ (C) $\frac{1}{6} = 0.166666...$ (D) $\frac{1}{7} = 0.142857...$ (E) $\frac{1}{8} = 0.125$

The choice that is closest to 0.17 is $\frac{1}{6}$, or (C).

Answer: (C)

15. Let us try the integers 5, 6, 7, 8.
When 5 is divided by 4, the quotient is 1 and the remainder is 1.
When 6 is divided by 4, the quotient is 1 and the remainder is 2.
When 7 is divided by 4, the quotient is 1 and the remainder is 3.
When 8 is divided by 4, the quotient is 2 and the remainder is 0.
The sum of these remainders is $1 + 2 + 3 + 0 = 6$.
(When any four consecutive integers are chosen, one will have a remainder 1, one a remainder of 2, one a remainder of 3 and one a remainder of 0 when divided by 4.)

Answer: (A)

16. Suppose that the initial radius of the circle is 1. Then its initial area is $\pi(1)^2 = \pi$ and its initial circumference is $2\pi(1) = 2\pi$.
When the radius is tripled, the new radius is 3.
The new area is $\pi(3)^2 = 9\pi$ and the new circumference is $2\pi(3) = 6\pi$ so the area is 9 times as large and the circumference is 3 times as large.

ANSWER: (A)

17. Since each number of votes in this problem is a multiple of 1000, we consider the number of thousands of votes that each potential Idol received, to make the numbers easier with which to work.
There was a total of 5219 thousand votes cast.
Suppose that the winner received x thousand votes. Then his opponents received $x-22$, $x-30$ and $x-73$ thousand votes.
Equating the total numbers of thousand of votes,

$$\begin{aligned} x + (x-22) + (x-30) + (x-73) &= 5219 \\ 4x - 125 &= 5219 \\ 4x &= 5344 \\ x &= 1336 \end{aligned}$$

Therefore, the winner received 1 336 000 votes.

ANSWER: (D)

18. When the number n is doubled, $2n$ is obtained.
When y is added, $2n + y$ is obtained.
When this number is divided by 2, we obtain $\frac{1}{2}(2n+y) = n + \dfrac{y}{2}$.
When n is subtracted, $\dfrac{y}{2}$ is obtained.

ANSWER: (E)

19. To make a fraction as large as possible, we should make the numerator as large as possible and the denominator as small as possible.
Of the four numbers in the diagram, z is the largest and w is the smallest, so the largest possible fraction is $\dfrac{z}{w}$.

ANSWER: (E)

20. *Solution 1*
Lorri's 240 km trip to Waterloo at 120 km/h took $240 \div 120 = 2$ hours.
Lorri's 240 km trip home at 80 km/h took $240 \div 80 = 3$ hours.
In total, Lorri drove 480 km in 5 hours, for an average speed of $\dfrac{480}{5} = 96$ km/h.

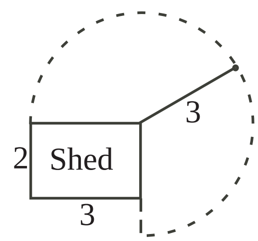
Solution 2
Lorri's 240 km trip to Waterloo at 120 km/h took $240 \div 120 = 2$ hours.
Lorri's 240 km trip home at 80 km/h took $240 \div 80 = 3$ hours.
Over the 5 hours that Lorri drove, her speeds were 120, 120, 80, 80, and 80, so her average speed was $\dfrac{120+120+80+80+80}{5} = \dfrac{480}{5} = 96$ km/h.

ANSWER: (B)

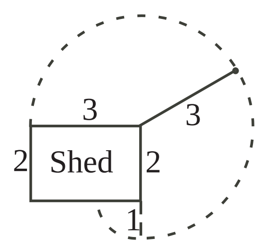
21. The area of rectangle $WXYZ$ is $10 \times 6 = 60$.
Since the shaded area is half of the total area of $WXYZ$, its area is $\frac{1}{2} \times 60 = 30$.
Since AD and WX are perpendicular, then the shaded area has four right angles, so is a rectangle.
Since square $ABCD$ has a side length of 6, then $DC = 6$.
Since the shaded area is 30, then $PD \times DC = 30$ or $PD \times 6 = 30$ or $PD = 5$.
Since $AD = 6$ and $PD = 5$, then $AP = 1$.

Answer: (A)

22. When Chuck has the leash extended to its full length, he can move in a 270° arc, or $\frac{3}{4}$ of a full circle about the point where the leash is attached. (He is blocked from going further by the shed.)



The area that he can play inside this circle is $\frac{3}{4}$ of the area of a full circle of radius 3, or $\frac{3}{4} \times \pi(3^2) = \frac{27}{4}\pi$.
When the leash is extended fully to the left, Chuck just reaches the top left corner of the shed, so can go no further.
When the leash is extended fully to the bottom, Chuck's leash extends 1 m below the length of the shed.
This means that Chuck can play in more area to the left.



This area is a 90° sector of a circle of radius 1, or $\frac{1}{4}$ of this circle. So this additional area is $\frac{1}{4} \times \pi(1^2) = \frac{1}{4}\pi$.
So the total area that Chuck has in which to play is $\frac{27}{4}\pi + \frac{1}{4}\pi = \frac{28}{4}\pi = 7\pi$.

Answer: (A)

23. *Solution 1*
Using \$5 bills, any amount of money that is a multiple of 5 (that is, ending in a 5 or a 0) can be made.
In order to get to \$207 from a multiple of 5 using only \$2 coins, the multiple of \$5 must end in a 5. (If it ended in a 0, adding \$2 coins would still give an amount of money that was an even integer, and so couldn't be \$207.)
Also, from any amount of money ending in a 5 that is less than \$207, enough \$2 coins can always be added to get to \$207.

The positive multiples of 5 ending in a 5 that are less than 207 are 5, 15, 25, ..., 195, 205. An easy way to count the numbers in this list is to remove the units digits (that is, the 5s) leaving 0, 1, 2, ..., 19, 20; there are 21 numbers in this list.
These are the only 21 multiples of 5 from which we can use \$2 coins to get to \$207. So there are 21 different ways to make \$207.

Solution 2
We are told that 1 \$2 coin and 41 \$5 bills make \$207. We cannot use fewer \$2 coins, since 0 \$2 coins would not work, so we can only use more \$2 coins.
To do this, we need to "make change" – that is, trade \$5 bills for \$2 coins. We cannot trade 1 \$5 bill for \$2 coins, since 5 is not even. But we can trade 2 \$5 bills for 5 \$2 coins, since each is worth \$10.
Making this trade once gets 6 \$2 coins and 39 \$5 bills.
Making this trade again get 11 \$2 coins and 37 \$5 bills.
We can continue to do this trade until we have only 1 \$5 bill remaining (and so \$202 in \$2 coins, or 101 coins).
So the possible numbers of \$5 bills are 41, 39, 37, ..., 3, 1. These are all of the odd numbers from 1 to 41. We can quickly count these to get 21 possible numbers of \$5 bills and so 21 possible ways to make \$207.

Answer: (E)

24. To get from $(2, 1)$ to $(12, 21)$, we go 10 units to the right and 20 units up, so we go 2 units up every time we go 1 unit to the right. This means that every time we move 1 unit to the right, we arrive at a lattice point. So the lattice points on this segment are

$$(2, 1), (3, 3), (4, 5), (5, 7), (6, 9), (7, 11), (8, 13), (9, 15), (10, 17), (11, 19), (12, 21)$$

(There cannot be more lattice points in between as we have covered all of the possible x-coordinates.)
To get from $(2, 1)$ to $(17, 6)$, we go 15 units to the right and 5 units up, so we go 3 units to the right every time we go 1 unit up. This means that every time we move 1 unit up, we arrive at a lattice point. So the lattice points on this segment are

$$(2, 1), (5, 2), (8, 3), (11, 4), (14, 5), (17, 6)$$

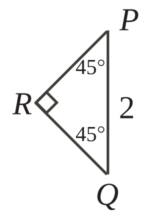
To get from $(12, 21)$ to $(17, 6)$, we go 5 units to the right and 15 units down, so we go 3 units down every time we go 1 unit to the right. This means that every time we move 1 unit to the right, we arrive at a lattice point. So the lattice points on this segment are

$$(12, 21), (13, 18), (14, 15), (15, 12), (16, 9), (17, 12)$$

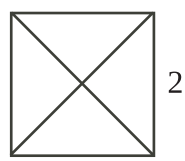
In total, there are $11 + 6 + 6 = 23$ points on our three lists. But 3 points (the 3 vertices of the triangle) have each been counted twice, so there are in fact $23 - 3 = 20$ different points on our lists.
Thus, there are 20 lattice points on the perimeter of the triangle.

Answer: (C)

25. To find the area of quadrilateral $DRQC$, we subtract the area of $\triangle PRQ$ from the area of $\triangle PDC$.
First, we calculate the area of $\triangle PDC$.
We know that $DC = AB = 5$ cm and that $\angle DCP = 90^\circ$.
When the paper is first folded, PC is parallel to AB and lies across the entire width of the paper, so $PC = AB = 5$ cm.
Therefore, the area of $\triangle PDC$ is $\frac{1}{2} \times 5 \times 5 = \frac{25}{2} = 12.5$ cm^2.
Next, we calculate the area of $\triangle PRQ$.
We know that $\triangle PDC$ has $PC = 5$ cm, $\angle PCD = 90^\circ$, and is isosceles with $PC = CD$.
Thus, $\angle DPC = 45^\circ$.
Similarly, $\triangle ABQ$ has $AB = BQ = 5$ cm and $\angle BQA = 45^\circ$.
Therefore, since $BC = 8$ cm and $PB = BC - PC$, then $PB = 3$ cm. Similarly, $AC = 3$ cm.
Since $PQ = BC - BP - QC$, then $PQ = 2$ cm.
Also, $\angle RPQ = \angle DPC = 45^\circ$ and $\angle RQP = \angle BQA = 45^\circ$.



Using four of these triangles, we can create a square of side length 2 cm (thus area 4 cm^2).



The area of one of these triangles (for example, $\triangle PRQ$) is $\frac{1}{4}$ of the area of the square, or 1 cm^2.
So the area of quadrilateral $DRQC$ is therefore $12.5 - 1 = 11.5$ cm^2.

Answer: (D)