Canadian Mathematics Competition

An activity of the Centre for Education in Mathematics and Computing, University of Waterloo, Waterloo, Ontario

2007 Fermat Contest

(Grade 11)

Tuesday, February 20, 2007

Solutions

1. Calculating, $\dfrac{36-12}{12-4} = \dfrac{24}{8} = 3$.

   Answer: (E)

2. Since $7x = 28$, then $x = 4$.
   Since $x + w = 9$ and $x = 4$, then $w = 5$.
   Since $x = 4$ and $w = 5$, then $xw = 20$.

   Answer: (B)

3. To determine which of the fractions are the largest and smallest, we write each using a common denominator of 16 as $\frac{12}{16}$, $\frac{14}{16}$, $\frac{13}{16}$, and $\frac{8}{16}$.
   Therefore, the largest is $\frac{7}{8} = \frac{14}{16}$ and the smallest is $\frac{1}{2} = \frac{8}{16}$.
   The difference between these two fractions is $\frac{14}{16} - \frac{8}{16} = \frac{6}{16} = \frac{3}{8}$.

   Answer: (A)

4. When $x = -5$, we have $-2x^2 + \dfrac{5}{x} = -2(-5)^2 + \dfrac{5}{-5} = -2(25) + (-1) = -50 - 1 = -51$.

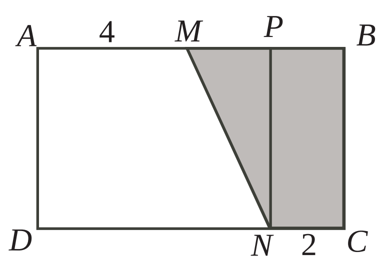
   Answer: (C)

5. By definition, $1^{-2} + 2^{-1} = \dfrac{1}{1^2} + \dfrac{1}{2^1} = \dfrac{1}{1} + \dfrac{1}{2} = \dfrac{3}{2}$.

   Answer: (A)

6. *Solution 1*
   Since the area of rectangle $ABCD$ is 40 and $AB = 8$, then $BC = 5$.
   Therefore, $MBCN$ is a trapezoid with height 5 and parallel bases of lengths 4 and 2, so has area $\frac{1}{2}(5)(4+2) = 15$.

   Solution 2
   Since the area of rectangle $ABCD$ is 40 and $AB = 8$, then $BC = 5$.
   Draw a line from N to AB parallel to BC (and so perpendicular to AB) meeting AB at P.

   This line divides $MBCN$ into a rectangle $PBCN$ of width 2 and height 5, and a triangle MPN with base MP of length 2 and height PN of height 5.
   The area of $MBCN$ is sum of the areas of these two parts, or $2(5) + \frac{1}{2}(2)(5) = 10 + 5 = 15$.

   Answer: (A)

7. *Solution 1*
   If the sum of two positive integers is 9, the possible pairs are 1 and 8, 2 and 7, 3 and 6, and 4 and 5.
   Of these pairs, the only one in which each number is a divisor of 42 is 2 and 7.
   Since the three positive integers have a product of 42, then two of them must be 2 and 7, so

the third is $42 \div (2 \times 7) = 42 \div 14 = 3$.

Solution 2
The possible sets of three positive integers which multiply to give 42 are $\{1, 1, 42\}$, $\{1, 2, 21\}$, $\{1, 3, 14\}$, $\{1, 6, 7\}$, and $\{2, 3, 7\}$.
The only one of these sets that contains two integers which add to 9 is $\{2, 3, 7\}$.
Therefore, the third number must be 3.

ANSWER: (D)

8. Suppose that Ivan ran a distance of x km on Monday.
Then on Tuesday, he ran $2x$ km, on Wednesday, he ran x km, on Thursday, he ran $\frac{1}{2}x$ km, and on Friday he ran x km.
The shortest of any of his runs was on Thursday, so $\frac{1}{2}x = 5$ or $x = 10$.
Therefore, his runs were 10 km, 20 km, 10 km, 5 km, and 10 km, for a total of 55 km.

ANSWER: (A)

9. Since $\dfrac{1}{x+3} = 2$, then, taking reciprocals of both sides, $x + 3 = \frac{1}{2}$.
Since $x + 3 = \frac{1}{2}$, then $x + 5 = \frac{1}{2} + 2 = \frac{5}{2}$.
Since $x + 5 = \frac{5}{2}$, then $\dfrac{1}{x+5} = \dfrac{2}{5}$.
(Notice that we did not need to actually find the value of x.)

ANSWER: (C)

10. Phyllis pays \$20 for each of two DVDs and \$10 for the third DVD, so pays \$50 in total for 3 DVDs.
Since \$50 is the price of $2\frac{1}{2}$ DVDs, then she gets 3 DVDs for the price of $2\frac{1}{2}$, which is the same as getting 6 DVDs for the price of 5.

ANSWER: (E)

11. When a set of five numbers is listed in ascending order, the median of the set is the middle number in the list, or the third number in this case.
Since the median is 7 and x is the middle number, then $x = 7$.
Therefore, the list is 2, 5, 7, 10, y.
Since the mean of the five numbers is 8, then the sum of the numbers is $5 \times 8 = 40$.
Therefore, $2 + 5 + 7 + 10 + y = 40$ or $24 + y = 40$ or $y = 16$.

ANSWER: (A)

12. Since $\angle QSR = \angle QRS$, then $\triangle QSR$ is isosceles with $QS = QR$, so $QS = x$.
Since $\angle SPQ = 90^\circ$ and $\angle PQS = 60^\circ$, then $\angle PSQ = 30^\circ$, so $\triangle PQS$ is a 30°-60°-90° triangle.
Therefore, $QS = 2PQ$ using the ratios of sides in such a triangle.
Thus, $x = QS = 2(10) = 20$.

ANSWER: (B)

13. *Solution 1*
Accounting for all of the missing numbers, $M + N + P + Q + R = 1 + 4 + 5 + 6 + 7 = 23$.
To determine the sum $M + N + P + Q$, we can determine the value of R and subtract this from 23.
R cannot be 1, since $0 + 1 = 1$ is not prime. (If R was 1, we would thus have the sum of the

numbers at the ends of one of the edges not equal to a prime number.)
R cannot be 4, since $0+4=4$ is not prime.
R cannot be 6, since $0+6=6$ is not prime.
R cannot be 7, since $2+7=9$ is not prime.
By process of elimination, $R=5$, so $M+N+P+Q=23-5=18$.
(We will see in Solution 2 that we can fill in the rest of the numbers in a way that satisfies the requirements.)

Solution 2
The missing numbers are 1, 4, 5, 6, 7.
Since $Q+3$ must be a prime number, then Q must be 4 (since $1+3$, $5+3$, $6+3$, and $7+3$ are not prime).
Since $M+0$ and $M+4$ must both be prime numbers, then M must be 7 (since $1+0$, $5+4$ and $6+4$ are not prime).
Since $P+2$ and $P+4$ must both be prime numbers, then P must be 1 (since $5+4$ and $6+2$ are not prime).
Since $N+7$ and $N+1$ must both be prime numbers, then N must be 6 (since $5+7$ is not prime).
We can check that if $R=5$, then the requirement that the sum of the two numbers at the ends of each edge be a prime number is met.
Thus, $M+N+P+Q=7+6+1+4=18$.

ANSWER: (C)

14. When a is increased by 25%, the result is $1.25a$ or $\frac{5}{4}a$.
Thus, we would like $\frac{5}{4}a > 5b$ or $5a > 20b$ or $a > 4b$.
We would like to find the smallest possible positive integers a and b that satisfy this inequality. (If a and b are as small as possible, then their sum $a+b$ will be as small as possible.)
Since b is a positive integer, $4b \geq 4$, so since $a > 4b$, then $a > 4$.
Thus, the smallest possible value of a is 5 when $b=1$.
Since b cannot be smaller, then a cannot be smaller.
Therefore, the minimum possible value for $a+b$ is $5+1=6$.

ANSWER: (B)

15. *Solution 1*
Suppose x has digits pqr.
Since x can only have even digits, then p is 2, 4, 6 or 8 and each of q and r can be 0, 2, 4, 6 or 8.
When x is multiplied by 2, each digit is multiplied by 2, and some "carrying" may occur.
Note that $2\times 0=0$, $2\times 2=4$, $2\times 4=8$, $2\times 6=12$, and $2\times 8=16$.
So when $2x$ is calculated, each digit from x will initially produce a corresponding even digit in $2x$ and a carry of 0 or 1.
If the carry is 0, whether the next digit to the left in $2x$ is even or odd is not affected.
If the carry is 1, the next digit to the left in $2x$ will be changed from even to odd. (Note that the carry can never be made larger than 1 by extra carrying into a given digit.)
Therefore, no digit of 6 or 8 can appear in x, since if a 6 or 8 appears in x, an odd digit in $2x$ is guaranteed.
Also, digits of 0, 2 and 4 can occur in any position (except no 0 in the leading position) as they will always produce an even digit in $2x$.
Therefore, there are 2 possible values for p and 3 each for q and r, giving $2\times 3\times 3=18$ possible values for x.

Solution 2

Suppose x has digits pqr, so $x = 100p + 10q + r$.

Since x can only have even digits, then p is 2, 4, 6 or 8 and each of q and r can be 0, 2, 4, 6 or 8.

When x is multiplied by 2, each digit is multiplied by 2, and some "carrying" may occur.

Note that $2 \times 0 = 0$, $2 \times 2 = 4$, $2 \times 4 = 8$, $2 \times 6 = 12$, and $2 \times 8 = 16$.

Suppose $2p = 10A + a$, $2q = 10B + b$, $2r = 10C + c$, where A, a, B, b, C, c are all digits with A, B, C each 0 or 1 and a, b, c each even.

Then

$$\begin{aligned} 2x &= 2(100p + 10q + r) \\ &= 100(2p) + 10(2q) + 2r \\ &= 100(10A + a) + 10(10B + b) + 10C + c \\ &= 1000A + 100a + 100B + 10b + 10C + c \\ &= 1000A + 100(a + B) + 10(b + C) + c \end{aligned}$$

Since a, b, c are each at most 8 and A, B, C are each at most 1, then $a + B$ and $b + C$ are each at most 9, so each of A, $a + B$, $b + C$ and c are indeed digits.

For $2x$ to have even digits only, then A must be even (so must be 0), c must be even (which it is), and $a + B$ and $b + C$ must both be even.

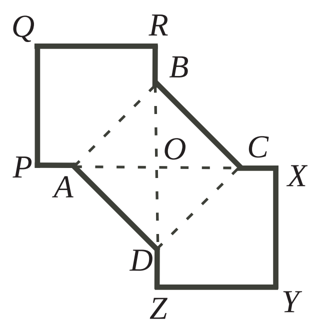
Since a and b are both even, this means that B and C must both be even, so must both be 0.

Since A, B and C are all 0, then none of p, q or r can be 6 or 8, and each can be 0, 2 or 4 (except $p \neq 0$).

Therefore, there are 2 possible values for p and 3 each for q and r, giving $2 \times 3 \times 3 = 18$ possible values for x.

Answer: (B)

16. We label the remaining vertices in the figure.



Since each of the squares has a side length of 3, then $PQ = QR = BC = XY = YZ = DA = 3$, so the perimeter of the figure equals $18 + AP + RB + CX + ZD$.

Since O is the centre of square $ABCD$, then $OA = OB = OC = OD$.

Since $OP = OR = OX = OZ$, then $AP = RB = CX = ZD$.

Therefore, the perimeter equals $18 + 4AP$.

Since O is centre of square $ABCD$, then $OA = OB$ and $\angle AOB = 90^\circ$, so $\triangle AOB$ is an isosceles right-angled triangle, so has angles 45°, 45° and 90°.

Therefore, $AO = \dfrac{AB}{\sqrt{2}} = \dfrac{3}{\sqrt{2}} = \dfrac{3\sqrt{2}}{2}$ and so $AP = OP - AO = 3 - \dfrac{3\sqrt{2}}{2}$.

Thus, the perimeter is $18 + 4\left(3 - \dfrac{3\sqrt{2}}{2}\right) = 18 + 12 - 6\sqrt{2} = 30 - 6\sqrt{2} \approx 21.515$.

Answer: (A)

17. *Solution 1*

Since $AB = BC$, then B lies on the perpendicular bisector of AC.
Since A has coordinates $(2,2)$ and C has coordinates $(8,4)$, then the midpoint of AC is $\left(\frac{1}{2}(2+8), \frac{1}{2}(4+2)\right) = (5,3)$ and the slope of AC is $\dfrac{4-2}{8-2} = \dfrac{1}{3}$.
Therefore, the slope of the perpendicular bisector is -3 (the negative reciprocal of $\frac{1}{3}$) and it passes through $(5,3)$, so has equation $y-3 = -3(x-5)$ or $y = -3x+18$.
The x-intercept of this line comes when y is set to 0; here, we obtain $x = 6$.
Therefore, since B is the point where the perpendicular bisector of AC crosses the x-axis, then the x-coordinate of B is 6.
(We can check that indeed if B has coordinates $(6,0)$, then AB and BC are perpendicular.)

Solution 2

Since $\triangle ABC$ is an isosceles right-angled triangle, then $\angle ABC = 90^\circ$, and so AB is perpendicular to BC.
Suppose B has coordinates $(b,0)$.
The slope of AB is $\dfrac{2-0}{2-b}$ and the slope of BC is $\dfrac{4-0}{8-b}$.
Since AB and BC are perpendicular, their slopes are negative reciprocals, so

$$\begin{aligned} \frac{2}{2-b} &= -\frac{8-b}{4} \\ -8 &= (2-b)(8-b) \\ -8 &= b^2 - 10b + 16 \\ b^2 - 10b + 24 &= 0 \\ (b-4)(b-6) &= 0 \end{aligned}$$

and so $b = 4$ or $b = 6$.
We must determine which value of b gives $AB = BC$ (since we have already used the perpendicularity).
If $b = 4$, then $AB = \sqrt{(4-2)^2 + (0-2)^2} = \sqrt{8}$ and $BC = \sqrt{(8-4)^2 + (4-0)^2} = \sqrt{32}$ and so $AB \neq BC$.
Therefore, the x-coordinate of B must be 6.
(We can check that, in this case, AB does equal BC.)

Solution 3

To go from A to C, we go 6 units right and 2 units up.
Suppose that to go from A to B, we go p units right and q units down, were $p, q > 0$.
Since BC is equal and perpendicular to AB, then to go from B to C, we must go q units right and p units up.
(We can see this by looking at the slopes of segments AB and BC.)
Therefore, to get from A to C through B, we go $p+q$ units right and $q-p$ units up, so $p+q = 6$ and $q-p = 2$, as the result is the same as from going directly to C from A.
Since $p+q = 6$ and $q-p = 2$, then $2q = 8$ (adding the equations), so $q = 4$, and so $p = 2$.
Since A has coordinates $(2,2)$, then B has coordinates $(6,0)$ which lies on the x-axis as required.

ANSWER: (D)

18. Suppose that Alphonso and Karen each start with n apples.
After Karen gives 12 apples to Alphonso, Karen has $n-12$ apples and Alphonso has $n+12$

apples.
After Karen gives half of her remaining apples (that is, $\frac{1}{2}(n-12)$ apples) to Alphonso, she has $\frac{1}{2}(n-12)=\frac{1}{2}n-6$ apples and Alphonso has $n+12+\frac{1}{2}(n-12)=\frac{3}{2}n+6$ apples.
Since Alphonso now has four times as many as Karen, $4(\frac{1}{2}n-6)=\frac{3}{2}n+6$ or $2n-24=\frac{3}{2}n+6$ so $\frac{1}{2}n=30$ or $n=60$.
This means that Karen now has $\frac{1}{2}(60-12)=24$ apples.

Answer: (B)

19. In this problem, we use the *Triangle Inequality*, which says that in any triangle, the length of one side is less than the sum of the lengths of the other two sides. (For example, in $\triangle ABC$, $AC<AB+BC=19$.) The Triangle Inequality is true because the shortest distance between any two points is a straight line, so any other route (such as travelling along the other two sides) must be longer.
In $\triangle ABC$, $AC<AB+BC=19$.
In $\triangle ACD$, $DC<DA+AC$ or $19<5+AC$ or $AC>14$.
Of the given choices, only 15 lies between 14 and 19, so 15 must be the answer. (We can check that if $AC=15$, each of these two triangles can be constructed.)

Answer: (D)

20. *Solution 1*
A specific parabola that has this shape (that is, opening downwards and with a negative x-intercept that is more negative than the positive x-intercept is positive) is
$y=-(x+2)(x-1)=-x^2-x+2$.
In this parabola, $a=-1$, $b=-1$, $c=2$.
Using these values for a, b and c, only $c-a$ is positive.
Since we are looking for the possibility which is positive no matter which parabola is used, then $c-a$ must be the answer.

Solution 2
Since the parabola in the diagram opens downwards, then $a<0$. This tells us that (A) is not correct and that ab^2 is negative so (C) is not correct.
The y-intercept of the parabola $y=ax^2+bx+c$ is $y=c$. (This comes from setting $x=0$.)
Since the y-intercept is positive, then $c>0$.
This tells us that $c-a$ is positive (since a is negative) so must be the correct answer, since there is only one correct answer.
(We could check that b is negative using the fact that the x-coordinate of the vertex is negative and so both bc and $b-c$ must be negative.)

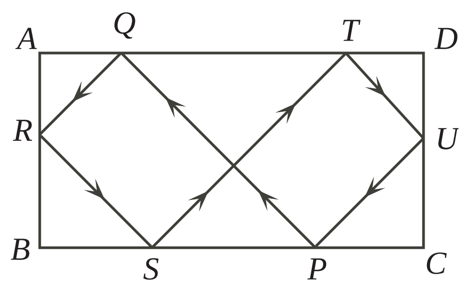
Answer: (E)

21. Since m is the third of the five integers, then the five integers are $m-2$, $m-1$, m, $m+1$, and $m+2$.
The sum of all five is thus $(m-2)+(m-1)+m+(m+1)+(m+2)=5m$ and the sum of the middle three is $(m-1)+m+(m+1)=3m$.
Therefore, we want to find the smallest integer m for which $3m$ is a perfect square and $5m$ is a perfect cube.
Consider writing m, $3m$ and $5m$ each as a product of prime numbers.
For $3m$ to be a perfect square, each prime must occur an even number of times in the product.
Thus, the prime 3 must occur an odd number of times in the product that represents m.
For $5m$ to be a perfect cube, the number of times that each prime occurs in the product must

be a multiple of 3. Thus, the prime 5 must occur a number of times which is one less than a multiple of 3 in the product that represents m.
Since both of the primes 3 and 5 are factors of m, then to minimize m, no other prime should occur.
For $3m$ to be a perfect square, 5 must occur an even number of times in $3m$.
For $5m$ to be a perfect cube, the number of times that 3 occurs in $5m$ is a multiple of 3.
Therefore, m is a number which contains an odd number of 3s and an even number of 5s (since $3m$ does), and at the same time contains 3 a number of times which is a multiple of 3 (since $5m$ does) and 5 a number of times that is 1 less than a multiple of 3.
To minimize m, m should contain as few 3s and 5s as possible, so should contain three 3s and two 5s, so $m = 3^3 5^2 = 675$.

Answer: (D)

22. Label the vertices of the rectangle $ABCD$ and the points of contact of the ball with the edges in order as P, Q, R, S, T, and U.



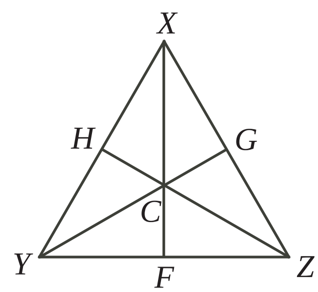
Note that the angle that each segment of the path makes with each side of the table is 45°.
Therefore, $PQ = \sqrt{2}AB$. Also, $QR = \sqrt{2}AR$ and $RS = \sqrt{2}RB$, so $QR+RS = \sqrt{2}(AR+RB) = \sqrt{2}AB$.
Similarly, $ST = \sqrt{2}CD$ and $TU + UP = \sqrt{2}CD$.
Since $AB = CD$, then $PQ + QR + RS + ST + TU + UP = 4\sqrt{2}AB$.
Since we know that the total length of the path is 7 m, then $AB = \frac{7}{4\sqrt{2}}$ m.

Also, $PQ = \sqrt{2} \times$ horizontal distance from P to Q, $QR = \sqrt{2}QA$ and $PU = \sqrt{2}PC$.
Therefore, $PQ + QR + PU = \sqrt{2}AD$ (since QA plus PC plus the horizontal distance from P to Q equals the length of the rectangle).
Similarly, $RS + ST + TU = \sqrt{2}BC = \sqrt{2}AD$.
So $PQ + QR + RS + ST + TU + UP = 2\sqrt{2}AD$ so $AD = \frac{7}{2\sqrt{2}}$.
Therefore, the perimeter of the table is $2AD + 2AB = \frac{7}{\sqrt{2}} + \frac{7}{2\sqrt{2}} \approx 7.425$, which is closest to 7.5 m.

Answer: (B)

23. Since the vertical distance from O to each of M, N and P is the same, then $OX = OY = OZ$ since the length of each wire is equal.
Let $x = OX = OY = OZ$.
Since each wire has total length 100, then $XM = YN = ZP = 100 - x$.
Therefore, M is a distance $100 - x$ below X. Since the total distance of M below the ceiling is 90, then the vertical distance from the ceiling to X (and thus to the plane of the triangle) is $90 - (100 - x) = x - 10$.
Let C be the centre of $\triangle XYZ$.

By symmetry, O is directly above C with $OC = x - 10$.
Also, $OX = x$.
By the Pythagorean Theorem, $OX^2 = OC^2 + XC^2$.
Therefore, we need to find the length of XC.
Draw altitudes from X, Y and Z to points F, G and H on the opposite sides. Note that C is the point of intersection of XF, YG and ZH.



Since $\triangle XYZ$ is equilateral, H is the midpoint of XY and XF is the angle bisector of $\angle ZXY$.
Therefore, $XH = 30$ (since $XY = 60$) and $\angle CXH = 30^\circ$.
$\triangle CXH$ is a 30°-60°-90° triangle, so $CX = \frac{2}{\sqrt{3}}XH = \frac{2}{\sqrt{3}}(30) = \frac{60\sqrt{3}}{3} = 20\sqrt{3}$.
Since $OX^2 = OC^2 + XC^2$, then

$$\begin{aligned} x^2 &= (x-10)^2 + (20\sqrt{3})^2 \\ x^2 &= x^2 - 20x + 100 + 1200 \\ 20x &= 1300 \\ x &= 65 \end{aligned}$$

Therefore, the distance between the triangle and the ceiling is $x - 10 = 55$ cm.

Answer: (D)

24. Suppose that the y-intercept of PR is b with $b > 0$.
Since PR has slope 1, then PR has x-intercept $-b$, so the coordinates of P are $(-b, 0)$.
Since P, Q and R lie on a line with $PQ = QR$, then the horizontal distance from P to Q equals the horizontal distance to Q to R. In other words, the difference between the x-coordinates of Q and P equals that of R and Q.
Since PR has slope 1 and y-intercept b, it has equation $y = x + b$.
We can determine the x-coordinates of Q and R by determining the points of intersection of $y = x^2$ and $y = x + b$, which we get by solving

$$\begin{aligned} x^2 &= x + b \\ x^2 - x - b &= 0 \\ x &= \frac{1 \pm \sqrt{1 - 4(1)(-b)}}{2} \qquad \text{(by the quadratic formula)} \\ x &= \frac{1 \pm \sqrt{1 + 4b}}{2} \end{aligned}$$

From the diagram, the x-coordinate of Q is $\dfrac{1 - \sqrt{1 + 4b}}{2}$ and the x-coordinate of R is $\dfrac{1 + \sqrt{1 + 4b}}{2}$.

From the given information,

$$\begin{aligned}
\frac{1-\sqrt{1+4b}}{2} - (-b) &= \frac{1+\sqrt{1+4b}}{2} - \frac{1-\sqrt{1+4b}}{2} \\
\frac{1-\sqrt{1+4b}}{2} + b &= \sqrt{1+4b} \\
1-\sqrt{1+4b}+2b &= 2\sqrt{1+4b} \\
1+2b &= 3\sqrt{1+4b} \\
(1+2b)^2 &= (3\sqrt{1+4b})^2 \\
1+4b+4b^2 &= 9(1+4b) \\
4b^2-32b-8 &= 0 \\
b^2-8b-2 &= 0 \\
b &= \frac{8 \pm \sqrt{8^2-4(1)(-2)}}{2} \\
b &= \frac{8 \pm \sqrt{72}}{2} \\
b &= 4 \pm 3\sqrt{2}
\end{aligned}$$

Since $b > 0$, then $b = 4 + 3\sqrt{2} \approx 8.243$.
(In this problem, it is in fact possible to determine which of the given answers is correct using a carefully drawn scale diagram.)

Answer: (C)

25. Consider arranging the $b+g$ balls in a row by first randomly choosing one ball placing it in the leftmost position, then randomly choosing another ball and placing it in the rightmost position, and then the choosing and placing the rest of the balls.

For both end balls to be black, the first ball must be black (the probability of this $\frac{b}{b+g}$) and then the second ball must be black (there are $b+g-1$ balls remaining of which $b-1$ is black so the probability is $\frac{b-1}{b+g-1}$), and the remaining balls can be placed in any way. Thus, the probability that both end balls are black is $\frac{b}{b+g} \cdot \frac{b-1}{b+g-1}$.

Similarly, the probability that both end balls are gold is $\frac{g}{b+g} \cdot \frac{g-1}{b+g-1}$.

Thus, the probability that both end balls are the same colour is

$$\frac{b}{b+g} \cdot \frac{b-1}{b+g-1} + \frac{g}{b+g} \cdot \frac{g-1}{b+g-1}$$

which we know should be equal to $\frac{1}{2}$.
Thus,

$$\begin{aligned}
\frac{b}{b+g} \cdot \frac{b-1}{b+g-1} + \frac{g}{b+g} \cdot \frac{g-1}{b+g-1} &= \frac{1}{2} \\
b(b-1)+g(g-1) &= \frac{1}{2}(b+g)(b+g-1) \\
2b^2-2b+2g^2-2g &= b^2+g^2+2bg-b-g \\
b^2-2bg+g^2 &= b+g \\
(g-b)^2 &= b+g
\end{aligned}$$

Set $g - b = k$. (Since $g \geq b$, then $k \geq 0$.)
Therefore, $b + g = k^2$.
Adding these two equations, we obtain $2g = k^2 + k$, so $g = \frac{1}{2}(k^2 + k) = \frac{1}{2}k(k+1)$.
Subtracting these two equations, we obtain $2b = k^2 - k$, so $b = \frac{1}{2}k(k-1)$. (This tells us that g and b are consecutive triangular numbers.)
Since $b \geq 4$, then $k(k-1) \geq 8$ so $k \geq 4$.
(If $k = 3$, the left side, which increases with k, equals 6; if $k = 4$, the left side equals 12.)
Since $g \leq 2007$, then $k(k+1) \leq 4014$ so $k \leq 62$.
(If $k = 63$, the left side, which increases with k, equals 4032; if $k = 62$, the left side equals 3906.)
Therefore, $4 \leq k \leq 62$, so there are $62 - 4 + 1 = 59$ possible values for k, and so 59 possible pairs (b, g).

Answer: (E)