**Canadian Mathematics Competition**

*An activity of the Centre for Education in Mathematics and Computing, University of Waterloo, Waterloo, Ontario*

# *Fermat Contest (Grade 11)*

***Wednesday, February 22, 2006***



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**Time:** 60 minutes



**Calculators are permitted**

**Instructions**

1. Do not open the Contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name, city/town, and province in the box in the upper left corner.
5. **Be certain that you code your name, age, sex, grade, and the Contest you are writing in the response form. Only those who do so can be counted as official contestants.**
6. This is a multiple-choice test. Each question is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. After making your choice, fill in the appropriate circle on the response form.
7. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
   There is *no penalty* for an incorrect answer.
   Each unanswered question is worth 2, to a maximum of 10 unanswered questions.
8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor tells you to begin, you will have *sixty* minutes of working time.

| Scoring: | There is *no penalty* for an incorrect answer.<br>Each unanswered question is worth 2, to a maximum of 10 unanswered questions. |
|---|---|

**Part A: Each correct answer is worth 5.**

1. The value of $\dfrac{1}{4 \times 5}$ is

   **(A)** 0.45 **(B)** 0.05 **(C)** 1.25 **(D)** 0.20 **(E)** 0.02

2. If $2x + 3x + 4x = 12 + 9 + 6$, then $x$ equals

   **(A)** 6 **(B)** 3 **(C)** 1 **(D)** $\frac{1}{3}$ **(E)** $10\frac{1}{2}$

3. The value of $\dfrac{4^3}{10^2 - 6^2}$ is

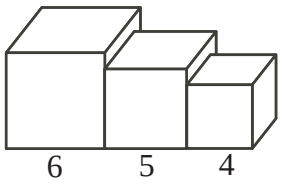
   **(A)** 1 **(B)** 0.5 **(C)** $-35.36$ **(D)** 1.5 **(E)** 4

4. The value of $\left(\sqrt{\sqrt{9} + \sqrt{1}}\right)^4$ is

   **(A)** $\sqrt{10}$ **(B)** 10 **(C)** 16 **(D)** 82 **(E)** 100

5. Three cubes have edges of lengths 4, 5 and 6.
   The average (mean) of their volumes is

   **(A)** 120 **(B)** 125 **(C)** 1125
   **(D)** 261 **(E)** 135




6. The regular price for a T-shirt is \$25 and the regular price for a pair of jeans is \$75. If the T-shirt is sold at a 30% discount and the jeans are sold at a 10% discount, then the total discount is

   **(A)** \$15 **(B)** \$20 **(C)** \$30 **(D)** \$36 **(E)** \$40

7. What is the smallest positive integer $p$ for which $\sqrt{2^3 \times 5 \times p}$ is an integer?

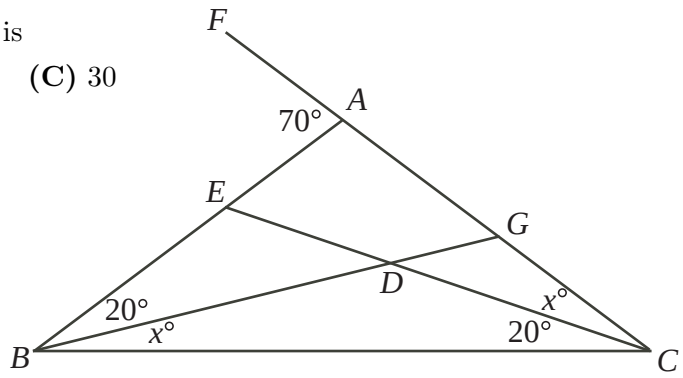
   **(A)** 2 **(B)** 5 **(C)** 10 **(D)** 1 **(E)** 20

8. If Corina had added the numbers $P$ and $Q$ correctly, the answer would have been 16. By mistake, she subtracted $Q$ from $P$. Her answer was 4. What is the value of $P$?

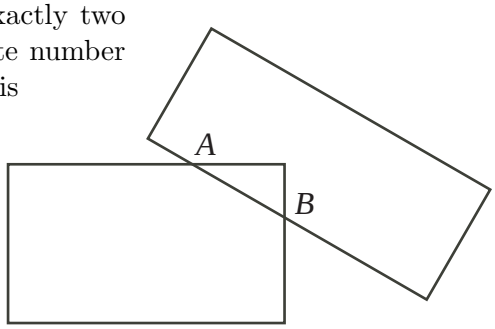
   **(A)** 4 **(B)** 5 **(C)** 8 **(D)** 10 **(E)** 16

9. In the diagram, the value of $x$ is

   **(A)** 15 **(B)** 20 **(C)** 30
   **(D)** 35 **(E)** 50



10. In the diagram, two rectangles intersect at exactly two points, $A$ and $B$. The maximum possible finite number of points of intersection of *any* two rectangles is




**(A)** 3 **(B)** 4 **(C)** 12 **(D)** 8 **(E)** 6

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**Part B: Each correct answer is worth 6.**

11. If $\frac{a}{b} = 3$ and $\frac{b}{c} = 2$, then the value of $\frac{a-b}{c-b}$ is

**(A)** $-4$ **(B)** $-\frac{1}{3}$ **(C)** $\frac{2}{3}$ **(D)** 2 **(E)** 6

12. If $\left(2^4\right)\left(3^6\right) = 9\left(6^x\right)$, what is the value of $x$?

**(A)** 2 **(B)** 3 **(C)** 4 **(D)** 216 **(E)** 8

13. In 2004, Gerry downloaded 200 songs. In 2005, Gerry downloaded 360 songs at a cost per song which was 32 cents less than in 2004. Gerry's *total* cost each year was the same. The cost of downloading the 360 songs in 2005 was

**(A)** \$144.00 **(B)** \$108.00 **(C)** \$80.00 **(D)** \$259.20 **(E)** \$72.00

14. If the system of equations

$$\begin{aligned} px + qy &= 8 \\ 3x - qy &= 38 \end{aligned}$$

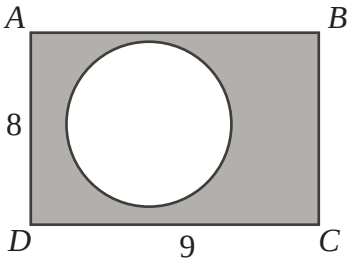
has the solution $(x, y) = (2, -4)$, then $p$ is equal to

**(A)** $-12$ **(B)** 20 **(C)** 8 **(D)** 40 **(E)** 21.5

15. The points $(5, 3)$ and $(1, -1)$ are plotted on a sheet of graph paper. The sheet of graph paper is folded along a line so that the point $(5, 3)$ lands on top of the point $(1, -1)$. The equation of the line that represents the fold is

**(A)** $y = -x + 1$ **(B)** $y = -x + 2$ **(C)** $y = -x + 3$
**(D)** $y = -x + 4$ **(E)** $y = -x + 5$

16. In the diagram, $ABCD$ is a rectangle. If the area of the circle is equal to the area of the shaded region, the radius of the circle is




**(A)** $\sqrt{\frac{6}{\pi}}$ **(B)** $\frac{6}{\pi}$ **(C)** $\frac{6}{\sqrt{\pi}}$
**(D)** $\sqrt{\frac{18}{\pi}}$ **(E)** $\frac{18}{\pi}$

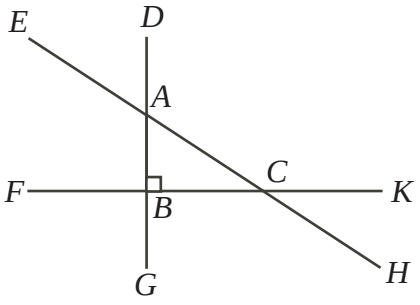
17. In seven term sequence, 5, $p$, $q$, 13, $r$, 40, $x$, each term after the third term is the sum of the preceding three terms. The value of $x$ is

**(A)** 21 **(B)** 61 **(C)** 67 **(D)** 74 **(E)** 80

18. The front wheel of Georgina's bicycle has a diameter of 0.75 metres. She cycled for 6 minutes at a speed of 24 kilometres per hour. The number of complete rotations that the wheel made during this time is closest to

**(A)** 610 **(B)** 1020 **(C)** 1360 **(D)** 1700 **(E)** 5430

19. In the diagram, $\triangle ABC$ is right-angled. Side $AB$ is extended in each direction to points $D$ and $G$ such that $DA = AB = BG$. Similarly, $BC$ is extended to points $F$ and $K$ so that $FB = BC = CK$, and $AC$ is extended to points $E$ and $H$ so that $EA = AC = CH$. The ratio of the area of the hexagon $DEFGHK$ to the area of $\triangle ABC$ is




**(A)** 4 : 1 **(B)** 7 : 1 **(C)** 9 : 1

**(D)** 16 : 1 **(E)** 13 : 1

20. A bag contains eight yellow marbles, seven red marbles, and five black marbles. Without looking in the bag, Igor removes $N$ marbles all at once. If he is to be sure that, no matter which choice of $N$ marbles he removes, there are at least four marbles of one colour and at least three marbles of another colour left in the bag, what is the maximum possible value of $N$?

**(A)** 6 **(B)** 7 **(C)** 8 **(D)** 9 **(E)** 10

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**Part C: Each correct answer is worth 8.**

21. For how many integers $n$, with $2 \leq n \leq 80$, is $\dfrac{(n-1)(n)(n+1)}{8}$ equal to an integer?

**(A)** 10 **(B)** 20 **(C)** 59 **(D)** 39 **(E)** 49

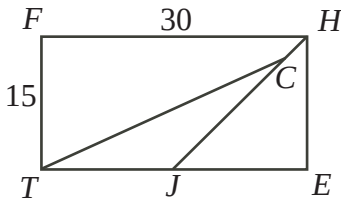
22. Quincy and Celine have to move 16 small boxes and 10 large boxes. The chart indicates the time that each person takes to move each type of box. They start moving the boxes at 9:00 a.m. The earliest time at which they can be finished moving all of the boxes is

| | Celine | Quincy |
|---|---|---|
| small box | 2 min. | 3 min. |
| large box | 6 min. | 5 min. |

**(A)** 9:41 a.m. **(B)** 9:42 a.m. **(C)** 9:43 a.m.

**(D)** 9:44 a.m. **(E)** 9:45 a.m.

23. Rectangle $TEHF$ has dimensions 15 m by 30 m, as shown. Tom the Cat begins at $T$, and Jerry the Mouse begins at $J$, the midpoint of $TE$. Jerry runs at 3 m/s in a straight line towards $H$. Tom starts at the same time as Jerry, and, running at 5 m/s in a straight line, arrives at point $C$ at the same time as Jerry. The time, in seconds, that it takes Tom to catch Jerry is closest to



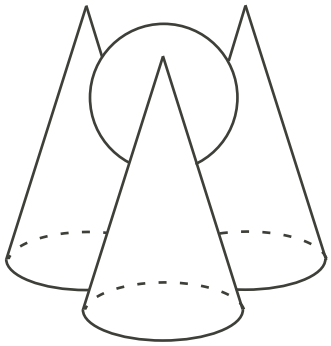

**(A)** 5.4 **(B)** 5.6 **(C)** 5.8

**(D)** 6.0 **(E)** 6.2

24. If $a$ and $b$ are positive integers such that $\frac{1}{a} + \frac{1}{2a} + \frac{1}{3a} = \frac{1}{b^2 - 2b}$, then the smallest possible value of $a + b$ is

**(A)** 8 **(B)** 6 **(C)** 96 **(D)** 10 **(E)** 50

25. Three identical cones each have a radius of 50 and a height of 120. The cones are placed so that their circular bases are touching each other. A sphere is placed so that it rests in the space created by the three cones, as shown. If the top of the sphere is level with the tops of the cones, then the radius of the sphere is closest to



**(A)** 38.9 **(B)** 38.7 **(C)** 38.1

**(D)** 38.5 **(E)** 38.3

## *Canadian Mathematics Competition*

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