2005 Galois Contest (Grade 10)

Wednesday, April 20, 2005

1. An *arithmetic sequence* is a sequence in which each term after the first is obtained from the previous term by adding a constant.
   For example, the sequence 2, 11, 20, 29, ... is an arithmetic sequence.
   (The "..." indicates that this sequence continues without ever ending.)

   (a) Find the 11th term in the arithmetic sequence 17, 22, 27, 32, ...

   (b) Explain why there is no number which occurs in each of the following arithmetic sequences:

   17, 22, 27, 32, ...
   13, 28, 43, 58, ...

   (c) Find a number between 400 and 420 which occurs in both of the following arithmetic sequences:

   17, 22, 27, 32, ...
   16, 22, 28, 34, ...

   Explain how you got your answer.

2. Emilia and Omar are playing a game in which they take turns placing numbered tiles on the grid shown.

Emilia starts the game with six tiles: $\boxed{1}$, $\boxed{2}$, $\boxed{3}$, $\boxed{4}$, $\boxed{5}$, and $\boxed{6}$.

Omar also starts the game with six tiles: $\boxed{1}$, $\boxed{2}$, $\boxed{3}$, $\boxed{4}$, $\boxed{5}$, and $\boxed{6}$.

Once a tile is placed, it cannot be moved.
After all of the tiles have been placed, Emilia scores one point for each row that has an even sum and one point for each column that has an even sum. Omar scores one point for each row that has an odd sum and one point for each column that has an odd sum. For example, if the game ends with the tiles placed as shown below, then Emilia will score 5 points and Omar 2 points.

3	1	2	4
5	5	2	4
1	3	6	6

   (a) In a game, after Omar has placed his second last tile, the grid appears as shown below. Starting with the partially completed game shown, give a final placement of tiles for which Omar scores more points than Emilia. (You do not have to give a strategy, simply fill in the final grid.)

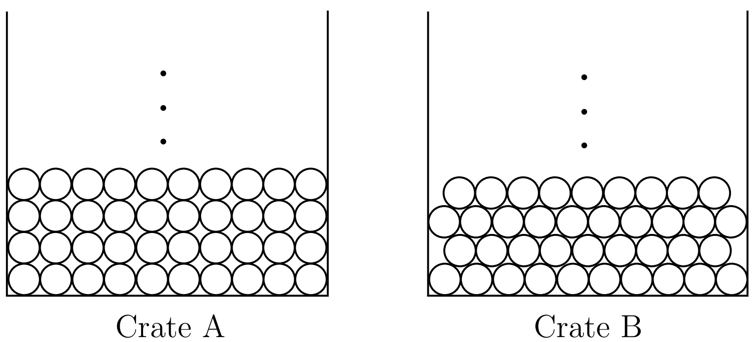
1	3		5
3	1	2	2
	4	4	

(b) Explain why it is impossible for Omar and Emilia to score the same number of points in any game.

(c) In the partially completed game shown below, it is Omar's turn to play and he has a 2 and a 5 still to place. Explain why Omar cannot score more points than Emilia, no matter where he places the 5.

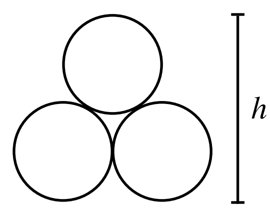
1		3	6
	5		4
3		1	6

3. Two identical rectangular crates are packed with cylindrical pipes, using different methods. Each pipe has diameter 10 cm. A side view of the first four rows of each of the two different methods of packing is shown below.



(a) If 200 pipes are packed in each of the two crates, how many rows of pipes are there in each crate? Explain your answer.

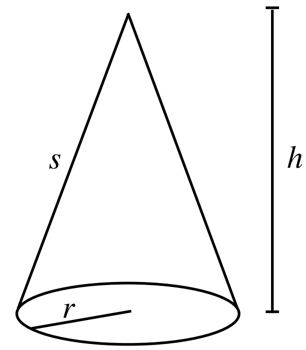
(b) Three pipes from Crate B are shown. Determine the height, h, of this pile of 3 pipes. Explain your answer.



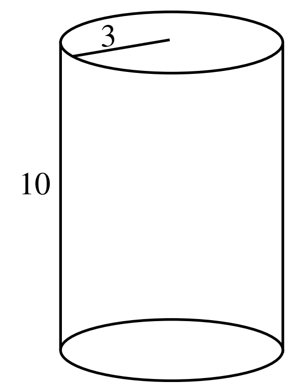
(c) After the crates have been packed with 200 pipes each, what is the difference in the total heights of the two packings? Explain your answer.

4. The volume of a sphere with radius r is $\frac{4}{3}\pi r^3$.

The *total* surface area of a cone with height h, slant height s, and radius r is $\pi r^2 + \pi r s$.



(a) A cylinder has a height of 10 and a radius of 3. Determine the *total* surface area, including the two ends, of the cylinder, and also determine the volume of the cylinder.



(b) A cone, a cylinder and a sphere all have radius r. The height of the cylinder is H and the height of the cone is h. The cylinder and the sphere have the same volume. The cone and the cylinder have the same total surface area. Prove that h and H cannot both be integers.