**Canadian
Mathematics
Competition**

*An activity of the Centre for Education
in Mathematics and Computing,
University of Waterloo, Waterloo, Ontario*

Fermat Contest (Grade 11)

Wednesday, February 23, 2005

C.M.C. Sponsors:

University of
Waterloo

**Deloitte
&Touche**
Chartered Accountants

C.M.C. Supporters:

Canadian Institute
of Actuaries

Great-West Life
ASSURANCE COMPANY

stronger communities together
GREAT-WEST LIFE | LONDON LIFE

Great West Life
and London Life

SYBASE
Sybase
*i*Anywhere
SOLUTIONS
*i*Anywhere Solutions

Time: 60 minutes   ©2004 Waterloo Mathematics Foundation
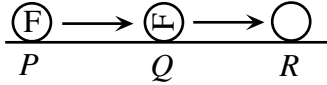**Calculators are permitted.**
Instructions

1. Do not open the Contest booklet until you are told to do so.

2. You may use rulers, compasses and paper for rough work.

3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.

4. On your response form, print your school name, city/town, and province in the box in the upper left corner.

5. **Be certain that you code your name, age, sex, grade, and the Contest you are writing in the response form. Only those who do so can be counted as official contestants.**

6. This is a multiple-choice test. Each question is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. After making your choice, fill in the appropriate circle on the response form.

7. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
   There is *no penalty* for an incorrect answer.
   Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

8. Diagrams are *not* drawn to scale. They are intended as aids only.

9. When your supervisor tells you to begin, you will have *sixty* minutes of working time.

Part A: Each correct answer is worth 5.

1. The value of $\dfrac{150 + (150 \div 10)}{15 - 5}$ is

   (A) 6 **(B)** 3 **(C)** 146 **(D)** 151.5 **(E)** 16.5

2. $\frac{1}{2} - \frac{1}{3} + \frac{3}{9}$ equals

   (A) $\frac{1}{4}$ **(B)** $\frac{1}{2}$ **(C)** $\frac{5}{18}$ **(D)** $\frac{1}{9}$ **(E)** 0

3. If $a = \frac{1}{2}$ and $b = \frac{2}{3}$, then $\dfrac{6a + 18b}{12a + 6b}$ equals

   (A) 9 **(B)** 7 **(C)** 10 **(D)** 6 **(E)** $\frac{3}{2}$

4. If $\sqrt{4 + 9 + x^2} = 7$, then a possible value for x is

   (A) 6 **(B)** 2 **(C)** 4 **(D)** 36 **(E)** 0

5. A Fermat coin rolls from P to Q to R, as shown. If the distance from P to Q is equal to the distance from Q to R, what is the orientation of the coin when it reaches R?

   (A) (F) **(B)** (Ⅎ) **(C)** (Ⅎ)

   (D) (Ⅎ) **(E)** (Ⅎ)

6. The sum of the first 2005 terms of the sequence 1, 2, 3, 4, 1, 2, 3, 4, . . . is

   (A) 5011 **(B)** 5110 **(C)** 5020 **(D)** 5010 **(E)** 501

7. In triangle ABC, $\angle A$ is 21° more than $\angle B$, and $\angle C$ is 36° more than $\angle B$. The size of $\angle B$ is

   (A) 20° **(B)** 41° **(C)** 62° **(D)** 46° **(E)** 56°

8. Seven children, each with the same birthday, were born in seven consecutive years. The sum of the ages of the youngest three children is 42. What is the sum of the ages of the oldest three?

   (A) 51 **(B)** 54 **(C)** 57 **(D)** 60 **(E)** 63

9. The lines $y = -2x + 8$ and $y = \frac{1}{2}x - 2$ meet at $(4, 0)$, as shown. The area of the triangle formed by these two lines and the line $x = -2$ is

    (A) 15   **(B)** 27   **(C)** 30
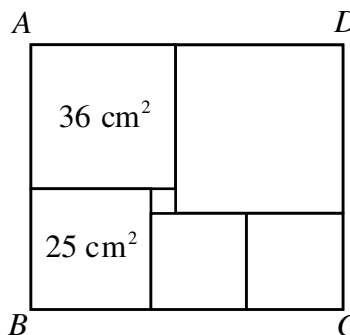
    (D) 36   **(E)** 45
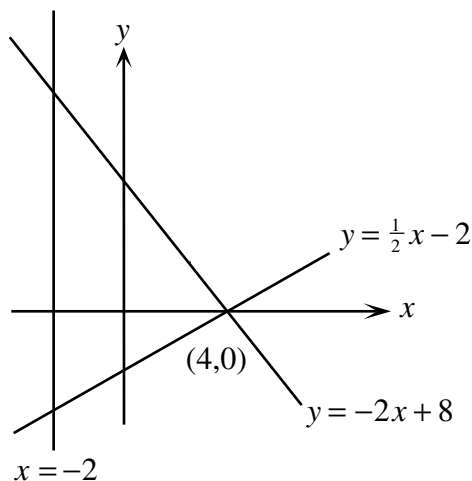
10. If 50% of P equals 20% of Q, then P, as a percent of Q, is

    (A) 60% **(B)** 250% **(C)** 40% **(D)** 20% **(E)** 30%


Part B: Each correct answer is worth 6.

11. Rectangle $ABCD$ is made up of six squares. The areas of two of the squares are shown. The perimeter of rectangle $ABCD$, in centimetres, is

    (A) 50   **(B)** 44   **(C)** 46

    (D) 52   **(E)** 48

12. Starting with the 2 in the centre, the number 2005 can be formed by moving from circle to circle only if the two circles are touching. How many different paths can be followed to form 2005?

    (A) 36   **(B)** 24   **(C)** 12

    (D) 18   **(E)** 6
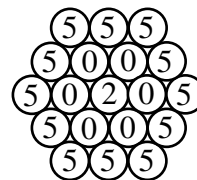
13. A circle is drawn so that no part of it lies outside a *regular* hexagon. If such a circle does not touch all six sides of the hexagon, what is the maximum number of sides that it could touch?

    (A) 1   **(B)** 2   **(C)** 3   **(D)** 4   **(E)** 5

14. The weight of a lioness is six times the weight of her female cub and four times the weight of her male cub. If the difference between the weights of the male and female cub is 14 kg, the weight of the lioness, in kg, is

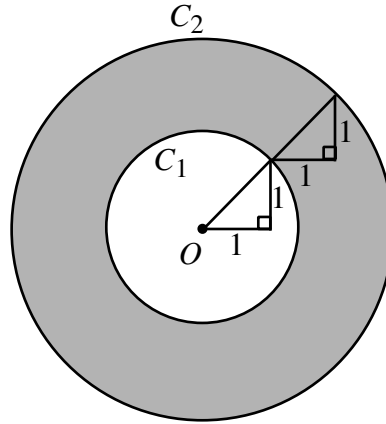   **(A)** 84   **(B)** 252   **(C)** 168   **(D)** 140   **(E)** 112

15. If $(x - 4)(5x + 2) = 0$, then the two possible values of $5x + 2$ are

    (A) -4 and $\frac{2}{5}$ **(B)** 0 and -18 **(C)** 0 and 22   **(D)** 0 and 4   **(E)** 4 and 22

16. In the diagram, circles C_1 and C_2 each have center O. The area of the shaded region is

(A) 2π (B) 3π (C) 4π

(D) 6π (E) 8π

17. A cylinder with radius 2 cm and height 8 cm is full of water. A second cylinder of radius 4 cm and height 8 cm is empty. If all of the water is poured from the first cylinder into the second cylinder, the depth of the water in the second cylinder will be

(A) 1 cm   (B) 2 cm   (C) 3 cm   (D) 4 cm   (E) 6 cm

18. A test has ten questions. Points are awarded as follows:
   • Each correct answer is worth 3 points.
   • Each unanswered question is worth 1 point.
   • Each incorrect answer is worth 0 points.
   A total score that is *not* possible is

(A) 11   (B) 13   (C) 17   (D) 23   (E) 29

19. Sam bicycles at 16 km/h and Chris bicycles at 24 km/h. At noon, Sam is 1 km north of Chris, and each begins to ride north. How many minutes will it take for Chris to catch Sam?

(A) $1\frac{1}{2}$ (B) $2\frac{1}{2}$ (C) $3\frac{3}{4}$ (D) $7\frac{1}{2}$ (E) 8

20. In triangle ABC, if $AB = AC = x + 1$ and $BC = 2x - 2$, where $x > 1$, then the area of the triangle is always equal to

(A) $(x - 1)\sqrt{2x^2 + 2}$ (B) $2(x - 1)$ (C) $\frac{1}{2}(x + 1)^2$

(D) $(x + 1)(x - 1)$ (E) $2(x - 1)\sqrt{x}$

Part C: Each correct answer is worth 8.

21. Four *different* numbers a, b, c, and d are chosen from the list -1, -2, -3, -4, and -5. The largest possible value for the expression $a^b + c^d$ is

(A) $\frac{5}{4}$ (B) $\frac{7}{8}$ (C) $\frac{31}{32}$ (D) $\frac{10}{9}$ (E) $\frac{26}{25}$

22. In the diagram, a semi-circle has diameter XY. Rectangle $PQRS$ is inscribed in the semi-circle with $PQ = 12$ and $QR = 28$. Square $STUV$ has T on RS, U on the semi-circle and V on XY. The area of $STUV$ is closest to

(A) 12   (B) 13   (C) 16

(D) 14   (E) 15
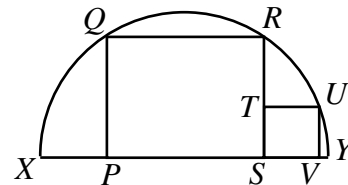
23. A solid cube of side length 4 cm is cut into two pieces by a plane that passed through the midpoints of six edges, as shown. To the nearest square centimetre, the surface area of each half cube created is

(A) 69   (B) 48   (C) 32

(D) 65   (E) 58
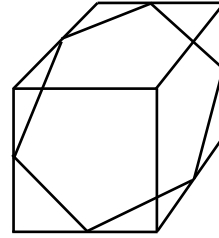
24. The arithmetic sequence a, $a+d$, $a+2d$, $a+3d$, ..., $a+(n-1)d$ has the following properties:

- When the first, third, and fifth, and so on terms are added, up to and including the last term, the sum is 320.
- When the first, fourth, seventh, and so on, terms are added, up to and including the last term, the sum is 224.

What is the sum of the whole sequence?

(A) 656   (B) 640   (C) 608   (D) 704   (E) 672

25. A *triline* is a line with the property that three times its slope is equal to the sum of its x-intercept and its y-intercept. For how many integers q with $1 \le q \le 10\,000$ is there at least one positive integer p so that there is exactly one triline through (p, q)?

(A) 60   (B) 57   (C) 58   (D) 61   (E) 59

N.B. This problem has been corrected from its original version with the addition of the underlined word "positive".

Canadian Mathematics Competition

For students...

Thank you for writing the 2005 Fermat Contest!
In 2004, more than 83 000 students around the world registered to write the Pascal, Cayley and Fermat Contests.

Encourage your teacher to register you for Hypatia Contest which will be written on April 20, 2005.
Visit our website

www.cemc.uwaterloo.ca

to find

- **More information about the Hypatia Contest**
- **Free copies of past Contests**
- **Workshops to help you prepare for future Contests**
- **Information about our publications for math enrichment and Contest preparation**
- **Information about careers in math**

For teachers...

Visit our website

www.cemc.uwaterloo.ca

to

- **Register your students for the Fryer, Galois and Hypatia Contests which will be written on April 20, 2005**
- **Learn about workshops and resources we offer for teachers**
- **Find your school results**