2004 Galois Contest (*Grade 10*)

Thursday, April 15, 2004

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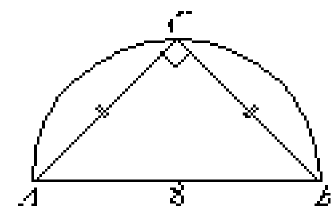
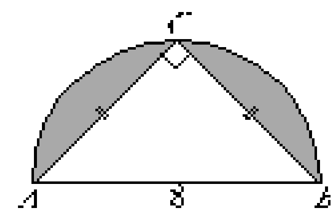
1. The Galois Group is giving out four types of prizes, valued at \$5, \$25, \$125 and \$625.
   (a) The Group gives out at least one of each type of prize. If five prizes are given out with a total value of \$905, how many of each type of prize is given out? Explain how you got your answer.
   (b) If the Group gives out at least one of each type of prize and five prizes in total, determine the other three possible total values it can give out. Explain how you got your answer.
   (c) There are two ways in which the Group could give away prizes totalling \$880 while making sure to give away at least one and at most six of each prize. Determine the two ways of doing this, and explain how you got your answer.

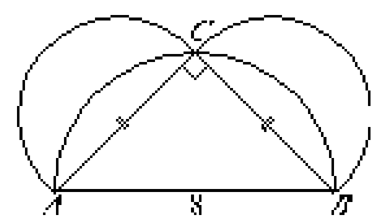
2. In the diagram, the semicircle has diameter $AB = 8$. Point C is on the semicircle so that triangle ABC is isosceles and right-angled.



   (a) Determine the area of triangle ABC.



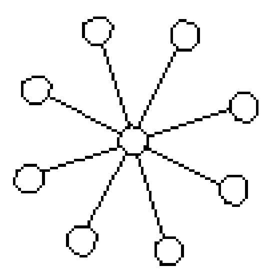
   (b) The two regions inside the semicircle but outside the triangle are shaded. Determine the total area of the two shaded regions.



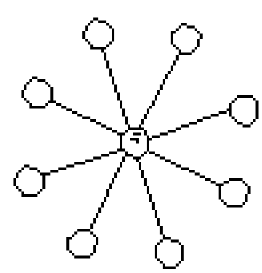
   (c) Semicircles are drawn on AC and CB, as shown. Show that:

$$(\text{Area of semicircle drawn on } AB) = (\text{Area of semicircle drawn on } AC) + (\text{Area of semicircle drawn on } CB)$$

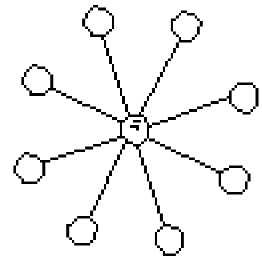
3. In "The Sun Game", two players take turns placing discs numbered 1 to 9 in the circles on the board. Each number may only be used once. The object of the game is to be the first to place a disc so that the sum of the 3 numbers along a line through the centre circle is 15.



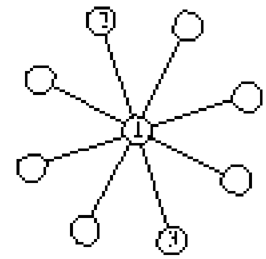
   (a) If Avril places a 5 in the centre circle and then Bob places a 3, explain how Avril can win on her next turn.



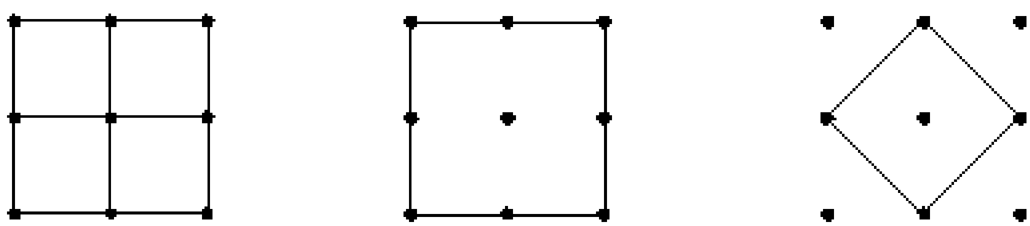
(b) If Avril starts by placing a 5 in the centre circle, show that whatever Bob does on his first turn, Avril can always win on her next turn.



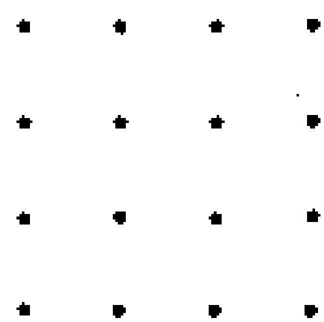
(c) If the game is in the position shown and Bob goes next, show that however Bob plays, Avril can win this game.



4. A 3 by 3 grid has dots spaced 1 unit apart both horizontally and vertically. Six squares of various side lengths can be formed with corners on the dots, as shown.



(a) Given a similar 4 by 4 grid of dots, there is a total of 20 squares of five different sizes that can be formed with corners on the dots. Draw one example of each size and indicate the number of squares there are of that size.



(b) In a 10 by 10 grid of dots, the number of squares that can be formed with side length $\sqrt{29}$ is two times the number of squares that can be formed with side length 7. Explain why this is true.

(c) Show that the total number of squares that can be formed in a 10 by 10 grid is
$1(9^2)+2(8^2)+3(7^2)+4(6^2)+5(5^2)+6(4^2)+7(3^2)+8(2^2)+9(1^2)$.