Canadian Mathematics Competition

An activity of The Centre for Education
in Mathematics and Computing,
University of Waterloo, Waterloo, Ontario

Pascal Contest (Grade 9)

Wednesday, February 19, 2003

Time: 1 hour

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**Calculators are permitted**.

Instructions

1. Do not open the contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name, city/town, and province in the box in the upper right corner.
5. **Be certain that you code your name, age, sex, grade, and the contest you are writing on the response form. Only those who do so can be counted as official contestants.**
6. This is a multiple-choice test. Each question is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. When you have decided on your choice, fill in the appropriate circle on the response form.
7. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
   There is *no penalty* for an incorrect answer.
   Each unanswered question is worth 2, to a maximum of 10 unanswered questions.
8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor instructs you to begin, you will have *sixty* minutes of working time.

Part A: Each correct answer is worth 5.

1. $\sqrt{169} - \sqrt{25}$ equals

   (A) 8   (B) 12   (C) 64   (D) 72   (E) 144

2. The missing number in the geometric sequence 2, 6, 18, 54, ____, 486 is

   (A) 72   (B) 90   (C) 108   (D) 162   (E) 216

3. The value of $\dfrac{6 + 6 \times 3 - 3}{3}$ is

   (A) 11   (B) 7   (C) 3   (D) 9   (E) 17

4. In the diagram, the value of x is

   (A) 40   (B) 60   (C) 100
   (D) 120   (E) 80
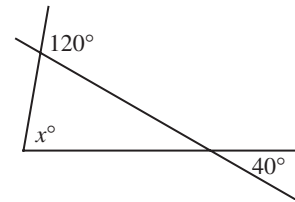
5. The value of $\dfrac{2^8}{8^2}$ is

   (A) $\frac{1}{16}$ (B) 8   (C) 4   (D) $\frac{1}{4}$ (E) 2

6. Which of the following is *not* equal to $\frac{18}{5}$?

   (A) $\dfrac{6^2}{10}$ (B) $\frac{1}{5}\left[6(3)\right]$ (C) $\dfrac{18+1}{5+1}$ (D) 3.6   (E) $\sqrt{\frac{324}{25}}$

7. In the diagram, the numbers 1, 2, 4, 5, 6, and 8 are substituted, in some order, for the letters A, B, C, D, E, and F, so that the number between and below two numbers is the positive difference between those two numbers. For example, the 7 in the third row is the positive difference between D and 9. Thus $D = 2$ because $9 - 2 = 7$. The value of $A + C$ is

   A 10 B C
       D 9 E
         7   F
          3

   (A) 7   (B) 12   (C) 13
   (D) 10   (E) 14
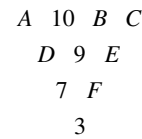
8. What is the area of rectangle $ABCD$?

   (A) 15   (B) 16   (C) 18

   (D) 30   (E) 9
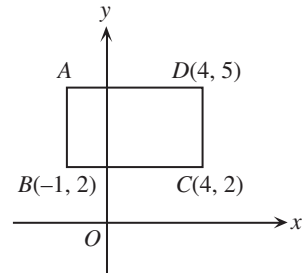
9. The largest prime number less than 30 that can be written as the sum of two primes is

   (A) 29   (B) 23   (C) 19   (D) 17   (E) 13

10. Which of the following numbers is the *largest*?

   (A) 3.2571   (B) $3.\overline{2571}$ (C) $3.2\overline{571}$ (D) $3.25\overline{71}$ (E) $3.257\overline{1}$

Part B: Each correct answer is worth 6.

11. If $x = 2$ and $y = -3$ satisfy the equation $2x^2 + kxy = 4$, then the value of k is

   (A) $\frac{2}{3}$ (B) 0   (C) $\frac{4}{3}$ (D) $-\frac{2}{3}$ (E) -2

12. At a math conference, the following exchange rates are used:

   $$1 \text{ calculator} = 100 \text{ rulers}$$
   $$10 \text{ rulers} = 30 \text{ compasses}$$
   $$25 \text{ compasses} = 50 \text{ protractors}$$

   How many protractors are equivalent to 1 calculator?

   (A) 400   (B) 600   (C) 300   (D) 500   (E) 200

13. In the diagram, each of the 15 small squares is going to be coloured. Any two squares that have a vertex in common or share a side must be a different colour. What is the *least* number of different colours needed?
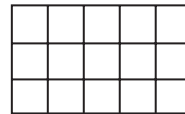
   (A) 3   (B) 4   (C) 5

   (D) 8   (E) 9

14. If x and y are positive integers and $x + y = 5$, then a possible value for $2x - y$ is

   (A) 3   (B) -3 (C) 2   (D) -2 (E) 0

15. In the diagram, square $ABCD$ is made up of 36 squares, each with side length 1. The area of the square $KLMN$, in square units, is

   (A) 12   (B) 16   (C) 18
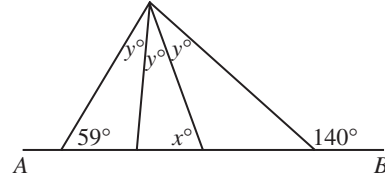
   (D) 20   (E) 25
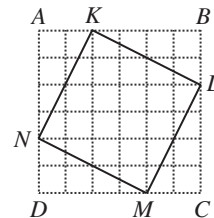
16. If n is any integer, $n+3$, $n-9$, $n-4$, $n+6$, and $n-1$ are also integers. If $n+3$, $n-9$, $n-4$, $n+6$, and $n-1$ are arranged from smallest to largest, the integer in the middle is

    (A) $n+3$ **(B)** $n-9$ **(C)** $n-4$ **(D)** $n+6$ **(E)** $n-1$

17. In the diagram, AB is a straight line. The value of x is

    (A) 67   **(B)** 59   **(C)** 62
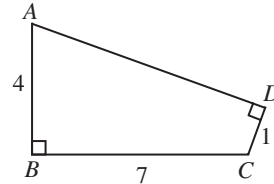   **(D)** 40   **(E)** 86

18. The average (mean) of a list of n numbers is 7. When the number -11 is added to the list, the new average is 6. What is the value of n?

    (A) 13   **(B)** 14   **(C)** 15   **(D)** 16   **(E)** 17

19. In the diagram, what is the area of quadrilateral $ABCD$?

    (A) 14   **(B)** 16   **(C)** 18
    (D) 20   **(E)** 28

20. The people of Evenland never use odd digits. Instead of counting 1, 2, 3, 4, 5, 6, an Evenlander counts 2, 4, 6, 8, 20, 22. What is an Evenlander's version of the integer 111?

    (A) 822   **(B)** 828   **(C)** 840   **(D)** 842   **(E)** 824


Part C: Each correct answer is worth 8.

21. A straight one-way city street has 8 consecutive traffic lights. Every light remains green for 1.5 minutes, yellow for 3 seconds, and red for 1.5 minutes. The lights are synchronized so that each light turns red 10 seconds after the preceding one turns red. What is the longest interval of time, in seconds, during which all 8 lights are green?

    (A) 10   **(B)** 15   **(C)** 20   **(D)** 25   **(E)** 30

22. In the diagram, two circles with centres A and B intersect at points P and Q so that $\angle PAQ = 60°$ and $\angle PBQ = 90°$. What is the ratio of the area of the circle with centre A to the area of the circle with centre B?

    (A) 3:1   **(B)** 3:2   **(C)** 4:3
    (D) 2:1   **(E)** 9:4
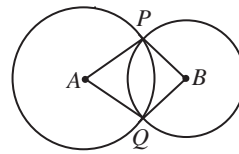
23. An escalator moves at a constant rate from one floor up to the next floor. Jack walks up 29 steps while travelling on the escalator between the floors. Jill takes twice as long to travel between the floors and walks up only 11 steps. When it is stopped, how many steps does the escalator have between the two floors?

    (A) 47   **(B)** 51   **(C)** 40   **(D)** 36   **(E)** 69

24. An artist wants to completely cover a rectangle with identically sized squares which do not overlap and do not extend beyond the edges of the rectangle. If the rectangle is $60\frac{1}{2}$ cm long and $47\frac{2}{3}$ cm wide, what is the minimum number of squares required?

(**A**) 429   (**B**) 858   (**C**) 1573   (**D**) 1716   (**E**) 5148

25. In the cube shown, L and K are midpoints of adjacent edges AD and AB. The perpendicular distance from F to the line segment LK is 10. What is the volume of the cube, to the nearest integer?

(**A**) 323   (**B**) 324   (**C**) 325

(**D**) 326   (**E**) 327

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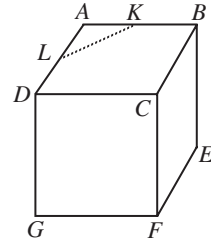
PUBLICATIONS

Students and parents who enjoy solving problems for fun and recreation may find the following publications of interest. They are an excellent resource for enrichment, problem solving and contest preparation.

Copies of Previous Canadian Mathematics Competitions
Copies of previous contests and solutions are available at no cost in both English and French at
http://www.cemc.uwaterloo.ca

Problems Problems Problems Books
Each volume is a collection of problems (multiple choice and full solution), grouped into 9 or more topics. Questions are selected from previous Canadian Mathematics Competition contests, and full solutions are provided for all questions. The price is $15. **(Available in English only.)**

Volume 1
• over 300 problems and full solutions
• 10 topics
• for students in Grades 9, 10, & 11
• French version of Volume 1 is available

Volume 2
• over 325 problems and full solutions
• 10 topics (different from Volume 1)
• for students in Grades 9, 10, & 11

Volume 3
• over 235 problems and full solutions
• 12 topics
• for senior high school students

Volume 4
• over 325 problems and full solutions
• 12 topics
• for students in Grades 7, 8, & 9

Volume 5
• over 200 problems and full solutions
• 9 topics (different from Volume 3)
• for senior high school students

Volume 6
• over 300 problems and full solutions
• 11 topics
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Volume 7
• over 300 problems and full solutions
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Problems and How To Solve Them - Volume 1
This book continues the collection of problems available for enrichment of students in grades 9, 10, and 11. Included for each of the eight chapters is a discussion on solving problems, with suggested approaches. There are more than 225 new problems, almost all from Canadian Mathematics Competitions, with complete solutions. The price is $20. **(Available in English only.)**

Orders should be addressed to: Canadian Mathematics Competition
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Include your name, address (with postal code), and telephone number.

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