



Canadian Mathematics Competition

An activity of The Centre for Education
in Mathematics and Computing,
University of Waterloo, Waterloo, Ontario

Cayley Contest (Grade 10)

Wednesday, February 19, 2003

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Time: 1 hour

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Calculators are permitted.

Instructions

- Do not open the contest booklet until you are told to do so.
- You may use rulers, compasses and paper for rough work.
- Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
- On your response form, print your school name, city/town, and province in the box in the upper right corner.
- Be certain that you code your name, age, sex, grade, and the contest you are writing on the response form. Only those who do so can be counted as official contestants.**
- This is a multiple-choice test. Each question is followed by five possible answers marked **A, B, C, D,** and **E.** Only one of these is correct. When you have decided on your choice, fill in the appropriate circle on the response form.
- Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
There is *no penalty* for an incorrect answer.
Each unanswered question is worth 2, to a maximum of 10 unanswered questions.
- Diagrams are *not* drawn to scale. They are intended as aids only.
- When your supervisor instructs you to begin, you will have *sixty* minutes of working time.



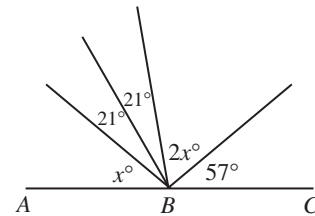
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Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

Part A: Each correct answer is worth 5.

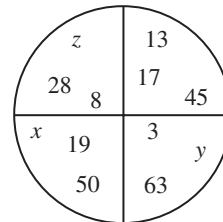
1. The value of $\frac{3-(-3)}{2-1}$ is
 (A) 2 (B) 0 (C) 3 (D) 6 (E) -3
2. $17^2 - 15^2$ equals
 (A) 8^2 (B) 2^2 (C) 4^2 (D) 7^2 (E) 6^2
3. The integer 42 is
 (A) an odd number (B) a prime number (C) a perfect cube
 (D) divisible by 7 (E) a perfect square
4. If 5% of a number is 8, what is 25% of the same number?
 (A) 40 (B) 0.1 (C) 320 (D) 10 (E) 200

5. The integer closest to the value of $\frac{3}{2} \times \frac{4}{9} + \frac{7}{2}$ is
 (A) 3 (B) 4 (C) 5 (D) 6 (E) 7

6. In the diagram, ABC is a straight line. The value of x is
 (A) 27 (B) 33 (C) 24
 (D) 87 (E) 81



7. In the diagram, the sum of the numbers in each quarter circle is the same. The value of $x + y + z$ is
 (A) 75 (B) 64 (C) 54
 (D) 171 (E) 300



8. An equilateral triangle has a side length of 20. If a square has the same perimeter as this triangle, the area of the square is
 (A) 25 (B) 400 (C) 225 (D) 60 (E) 100



9. If $\frac{1}{x + \frac{1}{5}} = \frac{5}{3}$, then x equals

- (A) $\frac{2}{5}$ (B) $\frac{4}{5}$ (C) $\frac{1}{5}$ (D) $-\frac{2}{5}$ (E) $-\frac{22}{5}$

10. There are 2 girls and 6 boys playing a game. How many additional girls must join the game so that $\frac{5}{8}$ of the players are girls?

- (A) 6 (B) 3 (C) 5 (D) 8 (E) 7

Part B: Each correct answer is worth 6.

11. Let $N = 10^3 + 10^4 + 10^5 + 10^6 + 10^7 + 10^8 + 10^9$. The sum of the digits of N is

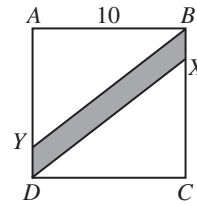
- (A) 12 (B) 1 (C) 6 (D) 9 (E) 7

12. The points $A(a, 1)$, $B(9, 0)$ and $C(-3, 4)$ lie on a straight line. The value of a is

- (A) 3 (B) $\frac{8}{3}$ (C) $\frac{7}{2}$ (D) 6 (E) $\frac{5}{2}$

13. In the diagram, $ABCD$ is a square with a side length of 10. If $AY = CX = 8$, the area of the shaded region is

- (A) 16 (B) 20 (C) 40
(D) 48 (E) 24

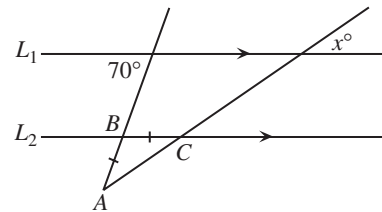


14. Carly takes three steps to walk the same distance as Jim walks in four steps. Each of Carly's steps covers 0.5 metres. How many metres does Jim travel in 24 steps?

- (A) 16 (B) 9 (C) 36 (D) 12 (E) 18

15. In the diagram, line L_1 is parallel to line L_2 and $BA = BC$. The value of x is

- (A) 35 (B) 30 (C) 37.5
(D) 45 (E) 40

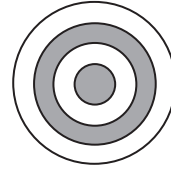


16. The value of $\frac{(4^{2003})(3^{2002})}{(6^{2002})(2^{2003})}$ is

- (A) 1 (B) 2 (C) 12 (D) 4 (E) $\frac{1}{2}$



17. In the diagram, the four circles have a common centre, and have radii of 1, 2, 3, and 4. The ratio of the area of the shaded regions to the area of the largest circle is



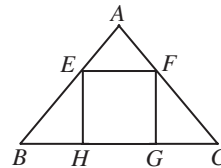
- (A) 5 : 8 (B) 1 : 4 (C) 7 : 16
(D) 1 : 2 (E) 3 : 8
18. If $496 = 2^m - 2^n$, where m and n are integers, then $m + n$ is equal to
(A) 13 (B) 9 (C) 4 (D) 14 (E) 5
19. The product of the digits of a four-digit number is 810. If none of the digits is repeated, the sum of the digits is
(A) 18 (B) 19 (C) 23 (D) 25 (E) 22
20. A car uses 8.4 litres of gas for every 100 km it is driven. A mechanic is able to modify the car's engine at a cost of \$400 so that it will only use 6.3 litres of gas per 100 km. The owner determines the minimum distance that she would have to drive to recover the cost of the modifications. If gas costs \$0.80 per litre, this distance, in kilometres, is between
(A) 10 000 and 14 000 (B) 14 000 and 18 000 (C) 18 000 and 22 000
(D) 22 000 and 26 000 (E) 26 000 and 30 000

Part C: Each correct answer is worth 8.

21. Troye and Daniella are running at constant speeds in opposite directions around a circular track. Troye completes one lap every 56 seconds and meets Daniella every 24 seconds. How many seconds does it take Daniella to complete one lap?

(A) 32 (B) 36 (C) 40 (D) 48 (E) 42

22. In the diagram, $\triangle ABC$ is isosceles with $AB = AC$ and $BC = 30$ cm. Square $EFGH$, which has a side length of 12 cm, is inscribed in $\triangle ABC$, as shown. The area of $\triangle AEF$, in cm^2 , is



(A) 27 (B) 54 (C) 51
(D) 48 (E) 60

23. A pyramid has a square base which has an area of 1440 cm^2 . Each of the pyramid's triangular faces is identical and each has an area of 840 cm^2 . The height of the pyramid, in cm, is

(A) $30\sqrt{2}$ (B) 40 (C) $20\sqrt{6}$ (D) $20\sqrt{3}$ (E) 30

24. In how many ways can a, b, c , and d be chosen from the set $\{0, 1, 2, \dots, 9\}$ so that $a < b < c < d$ and $a + b + c + d$ is a multiple of three?

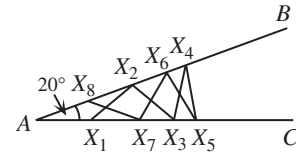
(A) 54 (B) 64 (C) 63 (D) 90 (E) 72

continued ...



25. $\angle BAC$ is said to be “laceable” if distinct points X_1, X_2, \dots, X_{2n} can be found so that

- X_{2k-1} is on AC for each value of k ,
- X_{2k} is on AB for each value of k , and
- $AX_1 = X_1X_2 = X_2X_3 = \dots = X_{2n-1}X_{2n} = X_{2n}A$.



For example, the angle 20° is laceable, as shown. The number of laceable acute angles, whose sizes in degrees are integers, is

- (A) 3 (B) 4 (C) 5
 (D) 6 (E) 7





PUBLICATIONS

Students and parents who enjoy solving problems for fun and recreation may find the following publications of interest. They are an excellent resource for enrichment, problem solving and contest preparation.

Copies of Previous Canadian Mathematics Competitions

Copies of previous contests and solutions are available at no cost in both English and French at <http://www.cemc.uwaterloo.ca>

Problems Problems Problems Books

Each volume is a collection of problems (multiple choice and full solution), grouped into 9 or more topics. Questions are selected from previous Canadian Mathematics Competition contests, and full solutions are provided for all questions. The price is \$15. (**Available in English only.**)

Volume 1

- over 300 problems and full solutions
- 10 topics
- for students in Grades 9, 10, & 11
- French version of Volume 1 is available

Volume 3

- over 235 problems and full solutions
- 12 topics
- for senior high school students

Volume 5

- over 200 problems and full solutions
- 9 topics (different from Volume 3)
- for senior high school students

Volume 7

- over 300 problems and full solutions
- 12 topics
- for students in Grades 9 and 10



Volume 2

- over 325 problems and full solutions
- 10 topics (different from Volume 1)
- for students in Grades 9, 10, & 11

Volume 4

- over 325 problems and full solutions
- 12 topics
- for students in Grades 7, 8, & 9

Volume 6

- over 300 problems and full solutions
- 11 topics
- for students in Grades 7, 8, & 9

Volume 8

- over 200 problems and full solutions
- 10 topics
- for students in Grades 11 and 12



Problems and How To Solve Them - Volume 1

This book continues the collection of problems available for enrichment of students in grades 9, 10, and 11. Included for each of the eight chapters is a discussion on solving problems, with suggested approaches. There are more than 225 new problems, almost all from Canadian Mathematics Competitions, with complete solutions. The price is \$20. (**Available in English only.**)

Orders should be addressed to: Canadian Mathematics Competition
Faculty of Mathematics, Room 5181
University of Waterloo
Waterloo, ON N2L 3G1

Include your name, address (with postal code), and telephone number.

Cheques or money orders in Canadian funds should be made payable to "Centre for Education in Mathematics and Computing". In Canada, add \$3.00 for the first item ordered for shipping and handling, plus \$1.00 for each subsequent item. No Provincial Sales Tax is required, but 7% GST must be added. Orders *outside of Canada ONLY*, add \$10.00 for the first item ordered for shipping and handling, plus \$2.00 for each subsequent item. **Prices for these publications will remain in effect until September 1, 2003.**

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