Canadian
Mathematics
Competition

An activity of The Centre for Education in Mathematics and Computing, University of Waterloo, Waterloo, Ontario

Fermat Contest (Grade 11)

Wednesday, February 20, 2002

Time: 1 hour

Calculators are permitted, providing they are non-programmable and without graphic displays.

Instructions

1. Do not open the contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name, city/town, and province in the box in the upper right corner.
5. **Be certain that you code your name, age, sex, grade, and the contest you are writing on the response form. Only those who do so can be counted as official contestants.**
6. This is a multiple-choice test. Each question is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. When you have decided on your choice, fill in the appropriate circle on the response form.
7. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
   There is *no penalty* for an incorrect answer.
   Each unanswered question is worth 2, to a maximum of 10 unanswered questions.
8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor instructs you to begin, you will have *sixty* minutes of working time.

Scoring:	There is *no penalty* for an incorrect answer. Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

Part A: Each correct answer is worth 5.

1. If $x = 3$, the numerical value of $5 - 2x^2$ is

   (**A**) –1 (**B**) 27 (**C**) –13 (**D**) – 31 (**E**) 3

2. $\dfrac{3^3 + 3}{2^2 + 2}$ is equal to

   (**A**) 3 (**B**) 6 (**C**) 2 (**D**) $\frac{3}{2}$ (**E**) 5

3. If it is now 9:04 a.m., in 56 hours the time will be

   (**A**) 9:04 a.m. (**B**) 5:04 p.m. (**C**) 5:04 a.m. (**D**) 1:04 p.m. (**E**) 1:04 a.m.

4. Which one of the following statements is **not** true?
   (**A**) 25 is a perfect square.
   (**B**) 31 is a prime number.
   (**C**) 3 is the smallest prime number.
   (**D**) 8 is a perfect cube.
   (**E**) 15 is the product of two prime numbers.

5. A rectangular picture of Pierre de Fermat, measuring 20 cm by 40 cm, is positioned as shown on a rectangular poster measuring 50 cm by 100 cm. What percentage of the area of the poster is covered by the picture?

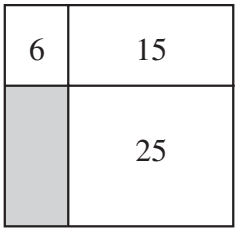
   

   (**A**) 24% (**B**) 16% (**C**) 20%
   (**D**) 25% (**E**) 40%

6. Gisa is taller than Henry but shorter than Justina. Ivan is taller than Katie but shorter than Gisa. The tallest of these five people is

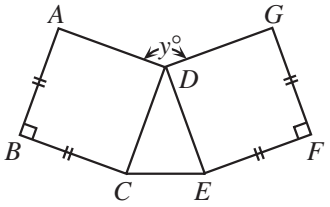
   (**A**) Gisa (**B**) Henry (**C**) Ivan (**D**) Justina (**E**) Katie

7. A rectangle is divided into four smaller rectangles. The areas of three of these rectangles are 6, 15 and 25, as shown. The area of the shaded rectangle is

   (**A**) 7 (**B**) 15 (**C**) 12
   (**D**) 16 (**E**) 10

8. In the diagram, $ABCD$ and $DEFG$ are squares with equal side lengths, and $\angle DCE = 70°$. The value of y is

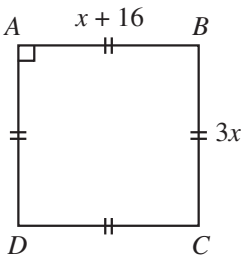


(**A**) 120 (**B**) 160 (**C**) 130
(**D**) 110 (**E**) 140

9. The numbers 1 through 20 are written on twenty golf balls, with one number on each ball. The golf balls are placed in a box, and one ball is drawn at random. If each ball is equally likely to be drawn, what is the probability that the number on the golf ball drawn is a multiple of 3?

(**A**) $\frac{3}{20}$ (**B**) $\frac{6}{20}$ (**C**) $\frac{10}{20}$ (**D**) $\frac{5}{20}$ (**E**) $\frac{1}{20}$

10. $ABCD$ is a square with $AB = x + 16$ and $BC = 3x$, as shown. The perimeter of $ABCD$ is



(**A**) 16 (**B**) 32 (**C**) 96
(**D**) 48 (**E**) 24

Part B: Each correct answer is worth 6.

11. A line passing through the points $(0,-2)$ and $(1,0)$ also passes through the point $(7,b)$. The numerical value of b is

(**A**) 12 (**B**) $\frac{9}{2}$ (**C**) 10 (**D**) 5 (**E**) 14

12. How many three-digit positive integers are perfect squares?

(**A**) 23 (**B**) 22 (**C**) 21 (**D**) 20 (**E**) 19

13. A "double-single" number is a three-digit number made up of two identical digits followed by a different digit. For example, 553 is a double-single number. How many double-single numbers are there between 100 and 1000?

(**A**) 81 (**B**) 18 (**C**) 72 (**D**) 64 (**E**) 90

14. The natural numbers from 1 to 2100 are entered sequentially in 7 columns, with the first 3 rows as shown. The number 2002 occurs in column m and row n. The value of $m+n$ is

	Column 1	Column 2	Column 3	Column 4	Column 5	Column 6	Column 7
Row 1	1	2	3	4	5	6	7
Row 2	8	9	10	11	12	13	14
Row 3	15	16	17	18	19	20	21
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮

(**A**) 290 (**B**) 291 (**C**) 292 (**D**) 293 (**E**) 294

15. In a sequence of positive numbers, each term after the first two terms is the sum of *all of the previous terms*. If the first term is a, the second term is 2, and the sixth term is 56, then the value of a is

(**A**) 1 (**B**) 2 (**C**) 3 (**D**) 4 (**E**) 5

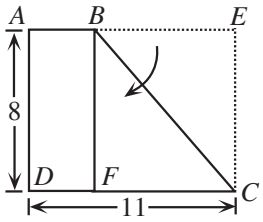
16. If $ac + ad + bc + bd = 68$ and $c + d = 4$, what is the value of $a + b + c + d$?

(**A**) 17 (**B**) 85 (**C**) 4 (**D**) 21 (**E**) 64

17. The average age of a group of 140 people is 24. If the average age of the males in the group is 21 and the average age of the females is 28, how many females are in the group?

(**A**) 90 (**B**) 80 (**C**) 70 (**D**) 60 (**E**) 50

18. A rectangular piece of paper $AECD$ has dimensions 8 cm by 11 cm. Corner E is folded onto point F, which lies on DC, as shown. The perimeter of trapezoid $ABCD$ is closest to

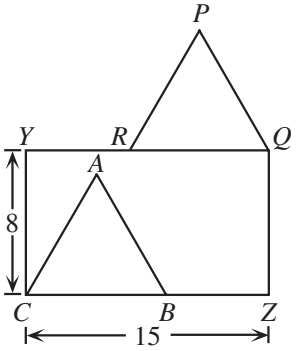


(**A**) 33.3 cm (**B**) 30.3 cm (**C**) 30.0 cm
(**D**) 41.3 cm (**E**) 35.6 cm

19. If $2^a 3^b = 8\left(6^{10}\right)$, where a and b are integers, then $b - a$ equals

(**A**) 0 (**B**) 23 (**C**) -13 (**D**) -7 (**E**) -3

20. In the diagram, $YQZC$ is a rectangle with $YC = 8$ and $CZ = 15$. Equilateral triangles ABC and PQR, each with side length 9, are positioned as shown with R and B on sides YQ and CZ, respectively. The length of AP is



(**A**) 10 (**B**) $\sqrt{117}$ (**C**) 9
(**D**) 8 (**E**) $\sqrt{72}$

Part C: Each correct answer is worth 8.

21. If $\sqrt{\frac{3}{1}\cdot\frac{5}{3}\cdot\frac{7}{5}\cdot\cdots\cdot\frac{2n+1}{2n-1}} = 9$, then the value of n is

(**A**) 38 (**B**) 1 (**C**) 40 (**D**) 4 (**E**) 39

22. The function $f(x)$ has the property that $f(x+y) = f(x) + f(y) + 2xy$, for all positive integers x and y. If $f(1) = 4$, then the numerical value of $f(8)$ is

(**A**) 72 (**B**) 84 (**C**) 88 (**D**) 64 (**E**) 80

continued ...

23. The integers from 1 to 9 are listed on a blackboard. If an additional m eights and k nines are added to the list, the average of all of the numbers in the list is 7.3. The value of $k+m$ is

(**A**) 24 (**B**) 21 (**C**) 11 (**D**) 31 (**E**) 89

24. A student has two open-topped cylindrical containers. (The walls of the two containers are thin enough so that their width can be ignored.) The larger container has a height of 20 cm, a radius of 6 cm and contains water to a depth of 17 cm. The smaller container has a height of 18 cm, a radius of 5 cm and is empty. The student slowly lowers the smaller container into the larger container, as shown in the cross-section of the cylinders in Figure 1. As the smaller container is lowered, the water first overflows out of the larger container (Figure 2) and then eventually pours into the smaller container. When the smaller container is resting on the bottom of the larger container, the depth of the water in the smaller container will be closest to

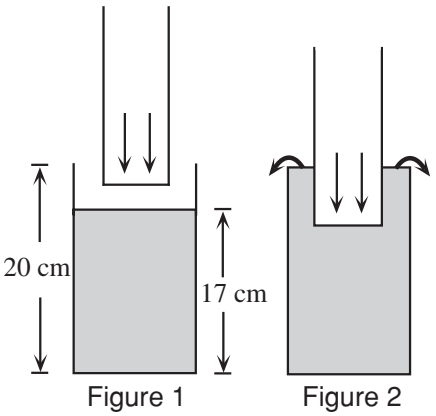


Figure 1 Figure 2

(**A**) 2.82 cm (**B**) 2.84 cm (**C**) 2.86 cm
(**D**) 2.88 cm (**E**) 2.90 cm

25. The lengths of all six edges of a tetrahedron are integers. The lengths of five of the edges are 14, 20, 40, 52, and 70. The number of possible lengths for the sixth edge is

(**A**) 9 (**B**) 3 (**C**) 4 (**D**) 5 (**E**) 6