Canadian Mathematics Competition

An activity of The Centre for Education
in Mathematics and Computing,
University of Waterloo, Waterloo, Ontario

Pascal Contest (Grade 9)

Wednesday, February 21, 2001

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Time: 1 hour

Calculators are permitted, providing they are non-programmable and without graphic displays.

Instructions

1. Do not open the contest booklet until you are told to do so.

2. You may use rulers, compasses and paper for rough work.

3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.

4. On your response form, print your school name, city/town, and province in the box in the upper right corner.

5. **Be certain that you code your name, age, sex, grade, and the contest you are writing on the response form. Only those who do so can be counted as official contestants.**

6. This is a multiple-choice test. Each question is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. When you have decided on your choice, fill in the appropriate circles on the response form.

7. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
   There is *no penalty* for an incorrect answer.
   Each unanswered question is worth 2, to a maximum of 20.

8. Diagrams are *not* drawn to scale. They are intended as aids only.

9. When your supervisor instructs you to begin, you will have *sixty* minutes of working time.

Part A: Each correct answer is worth 5.

1. The value of $\dfrac{5(6)-3(4)}{6+3}$ is

   (**A**) 1　　　　　(**B**) 2　　　　　(**C**) 6　　　　　(**D**) 12　　　　　(**E**) 31

2. When $12\,345\,678$ is divided by 10, the remainder is

   (**A**) 0　　　　　(**B**) 2　　　　　(**C**) 4　　　　　(**D**) 6　　　　　(**E**) 8

3. Evaluate $\dfrac{2^5-2^3}{2^2}$.

   (**A**) 6　　　　　(**B**) 1　　　　　(**C**) $\frac{1}{4}$　　　　　(**D**) 0　　　　　(**E**) 30

4. If $x=\frac{1}{4}$, which of the following has the largest value?

   (**A**) x　　　　　(**B**) x^2　　　　　(**C**) $\frac{1}{2}x$　　　　　(**D**) $\dfrac{1}{x}$　　　　　(**E**) $\sqrt{x}$

5. In the diagram, the value of x is

   (**A**) 100　　　　(**B**) 65　　　　(**C**) 80
   (**D**) 70　　　　(**E**) 50
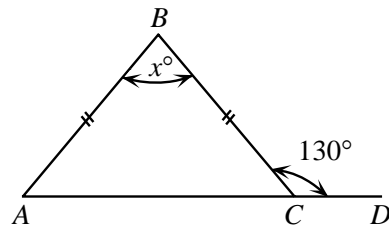
6. Anna's term mark was 80%. Her exam mark was 90%. In calculating her final mark, the term mark was given a weight of 70% and the exam mark a weight of 30%. What was her final mark?

   (**A**) 81%　　　　(**B**) 83%　　　　(**C**) 84%　　　　(**D**) 85%　　　　(**E**) 87%

7. The least value of x which makes $\dfrac{24}{x-4}$ an integer is

   (**A**) -44　　　　(**B**) -28　　　　(**C**) -20　　　　(**D**) -8　　　　(**E**) 0

8. The 50th term in the sequence $5,\ 6x,\ 7x^2,\ 8x^3,\ 9x^4,\ \ldots$ is

   (**A**) $54x^{49}$　　　　(**B**) $54x^{50}$　　　　(**C**) $45x^{50}$　　　　(**D**) $55x^{49}$　　　　(**E**) $46x^{51}$

9. The perimeter of $\triangle ABC$ is

(A) 23   (B) 40   (C) 42

(D) 46   (E) 60
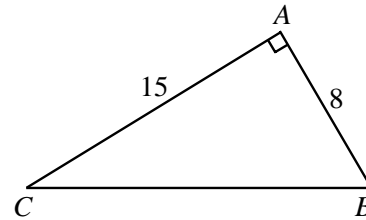
10. Dean scored a total of 252 points in 28 basketball games. Ruth played 10 fewer games than Dean. Her scoring average was 0.5 points per game higher than Dean's scoring average. How many points, in total, did Ruth score?

(A) 153   (B) 171   (C) 180   (D) 266   (E) 144

Part B: Each correct answer is worth 6.

11. Sahar walks at a constant rate for 10 minutes and then rests for 10 minutes. Which of these distance, d, versus time, t, graphs best represents his movement during these 20 minutes?

(A) d (B) d (C) d

(D) d (E) d
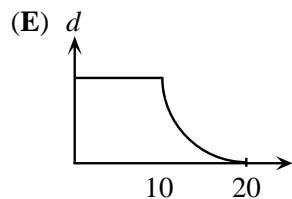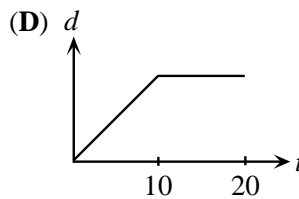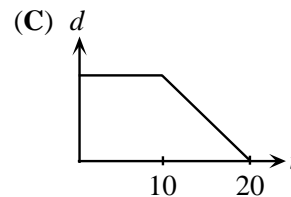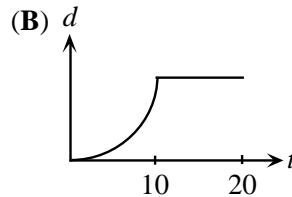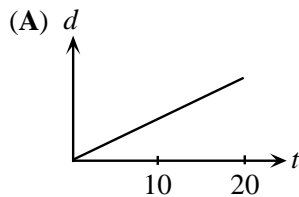
12. A bag contains 20 candies: 4 chocolate, 6 mint and 10 butterscotch. Candies are removed randomly from the bag and eaten. What is the minimum number of candies that must be removed to be *certain* that at least two candies of each flavour have been eaten?

(A) 6   (B) 10   (C) 12   (D) 16   (E) 18

13. Pierre celebrated his birthday on February 2, 2001. On that day, his age equalled the sum of the digits in the year in which he was born. In what year was Pierre born?

(A) 1987   (B) 1980   (C) 1979   (D) 1977   (E) 1971

14. Twenty tickets are numbered from one to twenty. One ticket is drawn at random with each ticket having an equal chance of selection. What is the probability that the ticket shows a number that is a multiple of 3 or 5?

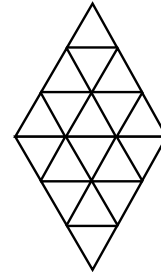(A) $\frac{3}{10}$ (B) $\frac{11}{20}$ (C) $\frac{2}{5}$ (D) $\frac{9}{20}$ (E) $\frac{1}{2}$

15. The line L crosses the x-axis at $(-8, 0)$. The area of the shaded region is 16. What is the slope of the line L?

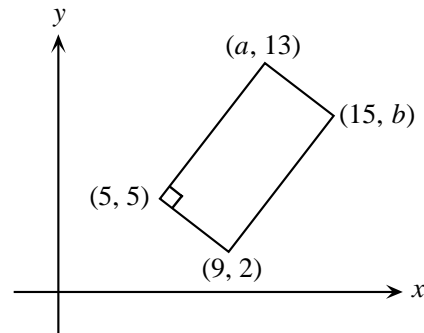(**A**) $\frac{1}{2}$ (**B**) 4   (**C**) $-\frac{1}{2}$

(**D**) 2   (**E**) -2
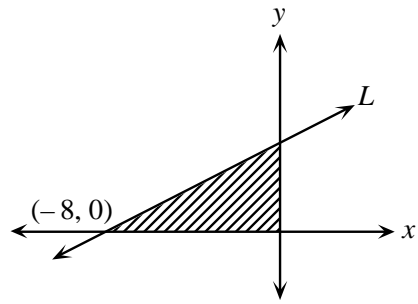
16. In the diagram, all triangles are equilateral. The total number of equilateral triangles of any size is

(**A**) 18   (**B**) 20   (**C**) 24

(**D**) 26   (**E**) 28

17. In the rectangle shown, the value of $a - b$ is

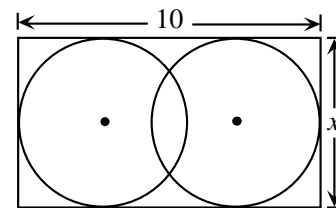(**A**) -3 (**B**) -1 (**C**) 0

(**D**) 3   (**E**) 1

18. The largest four-digit number whose digits add to 17 is 9800. The 5th largest four-digit number whose digits have a sum of 17 is

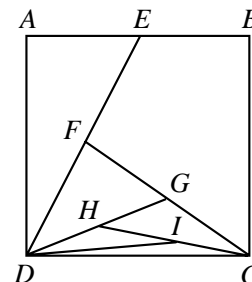(**A**) 9521   (**B**) 9620   (**C**) 9611   (**D**) 9602   (**E**) 9530

19. Two circles with equal radii are enclosed by a rectangle, as shown. The distance between their centres is $\dfrac{2x}{3}$. The value of x is

(**A**) $\frac{15}{4}$ (**B**) 5   (**C**) 6

(**D**) $\frac{60}{7}$ (**E**) $\frac{15}{2}$

20. Square $ABCD$ has an area of 4. E is the midpoint of AB. Similarly, $F, G, H,$ and I are the midpoints of $DE, CF, DG,$ and CH, respectively. The area of $\triangle IDC$ is

(**A**) $\frac{1}{4}$ (**B**) $\frac{1}{8}$ (**C**) $\frac{1}{16}$

(**D**) $\frac{1}{32}$ (**E**) $\frac{1}{64}$

Part C: Each correct answer is worth 8.

21. Cindy leaves school at the same time every day. If she cycles at 20 km/h, she arrives home at 4:30 in the afternoon. If she cycles at 10 km/h, she arrives home at 5:15 in the afternoon. At what speed, in km/h, must she cycle to arrive home at 5:00 in the afternoon?

    (A) $16\frac{2}{3}$ (B) 15   (C) $13\frac{1}{3}$ (D) 12   (E) $18\frac{3}{4}$

22. In the diagram, AB and BD are radii of a circle with centre B. The area of sector ABD is 2π, which is one-eighth of the area of the circle. The area of the shaded region is

    (A) $2\pi - 4$ (B) π (C) $2\pi - 2$

    (D) $2\pi - 4.5$ (E) $2\pi - 8$
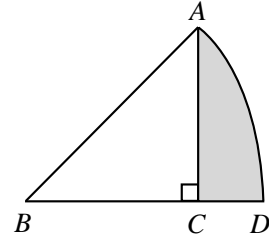
23. Five points are located on a line. When the ten distances between pairs of points are listed from smallest to largest, the list reads: 2, 4, 5, 7, 8, k, 13, 15, 17, 19. What is the value of k?

    (A) 11   (B) 9   (C) 13   (D) 10   (E) 12

24. A sealed bottle, which contains water, has been constructed by attaching a cylinder of radius 1 cm to a cylinder of radius 3 cm, as shown in Figure A. When the bottle is right side up, the height of the water inside is 20 cm, as shown in the cross-section of the bottle in Figure B. When the bottle is upside down, the height of the liquid is 28 cm, as shown in Figure C. What is the total height, in cm, of the bottle?

    Figure A   Figure B   Figure C
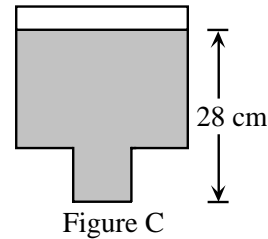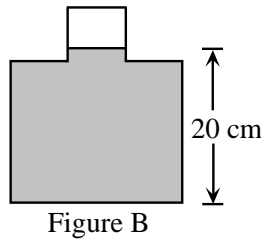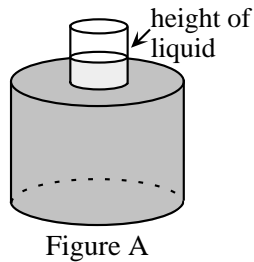
    (A) 29   (B) 30   (C) 31   (D) 32   (E) 48

25. A palindrome is a positive integer whose digits are the same when read forwards or backwards. For example, 2882 is a four-digit palindrome and 49194 is a five-digit palindrome. There are pairs of four-digit palindromes whose sum is a five-digit palindrome. One such pair is 2882 and 9339. How many such pairs are there?

    (A) 28   (B) 32   (C) 36   (D) 40   (E) 44