Canadian Mathematics Competition

An activity of *The Centre for Education in Mathematics and Computing,*
University of Waterloo, Waterloo, Ontario

Cayley Contest (Grade 10)

Wednesday, February 24, 1999

Time: 1 hour

Calculators are permitted, providing they are non-programmable and without graphic displays.

Instructions

1. Do not open the contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name, city/town, and province in the box in the upper right corner.
5. **Be certain that you code your name, age, sex, grade, and the contest you are writing on the response form. Only those who do so can be counted as official contestants.**
6. This is a multiple-choice test. Each question is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. When you have decided on your choice, fill in the appropriate circles on the response form.
7. Scoring: Each correct answer is worth 5 credits in Part A, 6 credits in Part B, and 8 credits in Part C.
   There is *no penalty* for an incorrect answer.
   Each unanswered question is worth 2 credits, to a maximum of 20 credits.
8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor instructs you to begin, you will have *sixty* minutes of working time.

Part A: Each question is worth 5 credits.

1. The value of $3^2 + 7^2 - 5^2$ is

    (A) 75   **(B)** 83   **(C)** 33   **(D)** 25   **(E)** 10

2. If 8 is added to the square of 5 the result is divisible by

    (A) 5   **(B)** 2   **(C)** 8   **(D)** 23   **(E)** 11

3. Today is Wednesday. What day of the week will it be 100 days from now?

    (A) Monday   **(B)** Tuesday   **(C)** Thursday   **(D)** Friday   **(E)** Saturday

4. The rectangle $PQRS$ is divided into six equal squares and shaded as shown. What fraction of $PQRS$ is shaded?
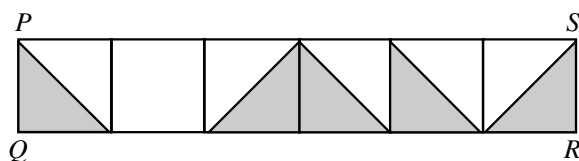
    (A) $\dfrac{1}{2}$ **(B)** $\dfrac{7}{12}$ **(C)** $\dfrac{5}{11}$ **(D)** $\dfrac{6}{11}$ **(E)** $\dfrac{5}{12}$

5. If $x = 4$ and $y = 3x$ and $z = 2y$, then the value of $y + z$ is

    (A) 12   **(B)** 20   **(C)** 40   **(D)** 24   **(E)** 36

6. In the diagram, the value of a is

    (A) 50   **(B)** 65   **(C)** 70
    (D) 105   **(E)** 110

7. In the diagram, AB and AC have equal lengths. What is the value of k?

    (A) -3 **(B)** -4 **(C)** -5
    (D) -7 **(E)** -8
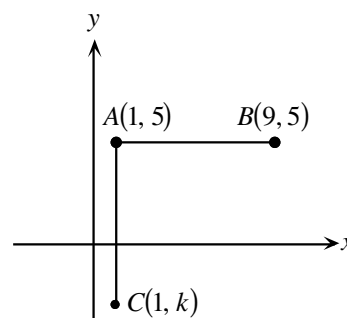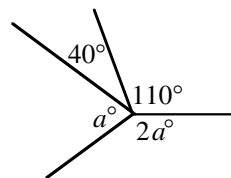
8. In the diagram, $AD < BC$. What is the perimeter of $ABCD$?
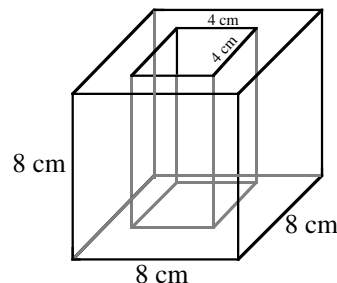
(A) 23   (B) 26   (C) 27
(D) 28   (E) 30
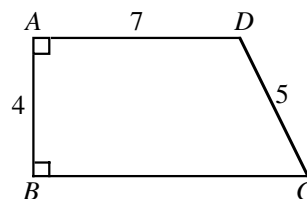
9. Three CD's are bought at an average cost of $15 each. If a fourth CD is purchased, the average cost becomes $16. What is the cost of the fourth CD?

(A) $16   (B) $17   (C) $18   (D) $19   (E) $20

10. An 8 cm cube has a 4 cm square hole cut through its centre, as shown. What is the remaining volume, in cm^3?

(A) 64   (B) 128   (C) 256
(D) 384   (E) 448

Part B: Each question is worth 6 credits.

11. The time on a digital clock is 5:55. How many minutes will pass before the clock next shows a time with all digits identical?

(A) 71   (B) 72   (C) 255   (D) 316   (E) 436

12. The numbers 49, 29, 9, 40, 22, 15, 53, 33, 13, 47 are grouped in pairs so that the sum of each pair is the same. Which number is paired with 15?

(A) 33   (B) 40   (C) 47   (D) 49   (E) 53

13. The units digit in the product $\left(5^2 + 1\right)\left(5^3 + 1\right)\left(5^{23} + 1\right)$ is

(A) 0   (B) 1   (C) 2   (D) 5   (E) 6

14. In an election for class president, 61 votes are cast by students who are voting to choose one of four candidates. Each student must vote for only one candidate. The candidate with the highest number of votes is the winner. The smallest number of votes the winner can receive is

(A) 15   (B) 16   (C) 21   (D) 30   (E) 31

15. A chocolate drink is 6% pure chocolate, by volume. If 10 litres of pure milk are added to 50 litres of this drink, the percent of chocolate in the new drink is

(A) 5   (B) 16   (C) 10   (D) 3   (E) 26

16. Three circles, each with a radius of 10 cm, are drawn tangent to each other so that their centres are all in a straight line. These circles are inscribed in a rectangle which is inscribed in another circle. The area of the largest circle is

   (A) 1000π (B) 1700π (C) 900π
   (D) 1600π (E) 1300π
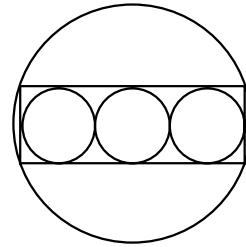
17. Let N be the smallest positive integer whose digits have a product of 2000. The sum of the digits of N is

   (A) 21   (B) 23   (C) 25   (D) 27   (E) 29

18. A cylindrical pail containing water drains into a cylindrical tub 40 cm across and 50 cm deep, while resting at an angle of 45° to the horizontal, as shown. How deep is the water in the tub when its level reaches the pail?
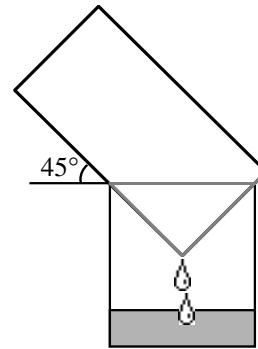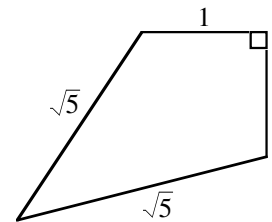
   (A) 10 cm   (B) 20 cm   (C) 30 cm
   (D) 35 cm   (E) 40 cm

19. A number is *Beprisque* if it is the only natural number between a prime number and a perfect square (e.g. 10 is Beprisque but 12 is not). The number of *two-digit* Beprisque numbers (including 10) is

   (A) 1   (B) 2   (C) 3   (D) 4   (E) 5

20. The area of the given quadrilateral is

   (A) $\dfrac{3}{2}$ (B) $\sqrt{5}$ (C) $\dfrac{1+\sqrt{10}}{2}$
   (D) 2   (E) 3

Part C: Each question is worth 8 credits.

21. A number is formed using the digits $1, 2, ..., 9$. Any digit can be used more than once, but adjacent digits cannot be the same. Once a pair of adjacent digits has occurred, that pair, in that order, cannot be used again. How many digits are in the largest such number?

   (A) 72   (B) 73   (C) 144   (D) 145   (E) 91

22. A main gas line runs through P and Q. From some point T on PQ, a supply line runs to a house at point M. A second supply line from T runs to a house at point N. What is the minimum total length of pipe required for the two supply lines?

M ←——105 m——→ P

130 m

←55 m—→ Q
N
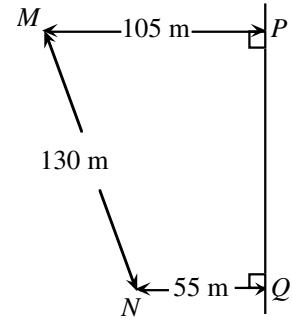
   (**A**) 200   (**B**) 202   (**C**) 198
   (**D**) 210   (**E**) 214

23. How many integers can be expressed as a sum of three distinct numbers chosen from the set $\{4, 7, 10, 13, ..., 46\}$?

   (**A**) 45   (**B**) 37   (**C**) 36   (**D**) 43   (**E**) 42

24. The sum of all values of x that satisfy the equation $\left(x^2 - 5x + 5\right)^{x^2+4x-60} = 1$ is

   (**A**) -4 (**B**) 3   (**C**) 1   (**D**) 5   (**E**) 6

25. If $a = 3^p$, $b = 3^q$, $c = 3^r$, and $d = 3^s$ and if p, q, r, and s are positive integers, determine the smallest value of $p+q+r+s$ such that $a^2 + b^3 + c^5 = d^7$.

   (**A**) 17   (**B**) 31   (**C**) 106   (**D**) 247   (**E**) 353