35th Anniversary
1963 – 1998

Canadian Mathematics Competition

An activity of The Centre for Education in Mathematics and Computing, University of Waterloo, Waterloo, Ontario

# *Fermat Contest* (Grade 11)

## Wednesday, February 18, 1998



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**Time:** 1 hour



**Calculators are permitted**, providing they are non-programmable and without graphic displays.

**Instructions**

1. Do not open the contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name, city/town, and province in the box in the upper right corner.
5. **Be certain that you code your name, age, sex, grade, and the contest you are writing on the response form. Only those who do so can be counted as official contestants.**
6. This is a multiple-choice test. Each question is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. When you have decided on your choice, fill in the appropriate circles on the response form.
7. Scoring: Each correct answer is worth 5 credits in Part A, 6 credits in Part B, and 8 credits in Part C.
   There is *no penalty* for an incorrect answer.
   Each unanswered question is worth 2 credits, to a maximum of 20 credits.
8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor instructs you to begin, you will have *sixty* minutes of working time.

Scoring: There is *no penalty* for an incorrect answer.
Each unanswered question is worth 2 credits, to a maximum of 20 credits.

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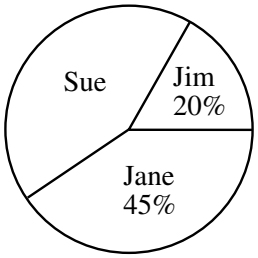
**Part A: Each question is worth 5 credits.**

1. The value of $\frac{1+2+3+4+5}{2+4+6+8+10}$ is

   **(A)** $\frac{1}{3}$ **(B)** 2.5 **(C)** $\frac{1}{2}$ **(D)** $\frac{11}{26}$ **(E)** $\frac{3}{8}$

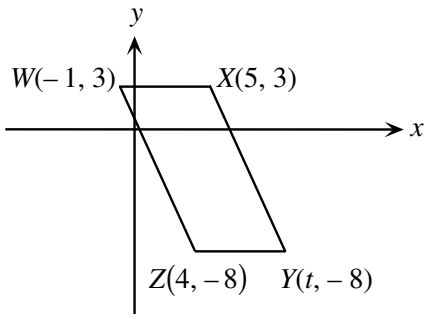
2. The pie chart shows a percentage breakdown of 1000 votes in a student election. How many votes did Sue receive?

   **(A)** 550 **(B)** 350 **(C)** 330
   **(D)** 450 **(E)** 935




3. If $WXYZ$ is a parallelogram, then $t$ equals

   **(A)** 8 **(B)** 9 **(C)** 10
   **(D)** 11 **(E)** 12




4. The product of two positive integers $p$ and $q$ is 100. What is the largest possible value of $p+q$?

   **(A)** 52 **(B)** 101 **(C)** 20 **(D)** 29 **(E)** 25

5. If $\otimes$ is a new operation defined as $p \otimes q = p^2 - 2q$, what is the value of $7 \otimes 3$?

   **(A)** 43 **(B)** 8 **(C)** 141 **(D)** 36 **(E)** 26

6. The value of $\frac{1}{3}$ of $6^{30}$ is

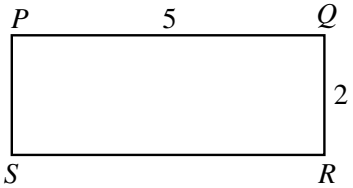
   **(A)** $6^{10}$ **(B)** $2^{30}$ **(C)** $2^{10}$ **(D)** $2 \times 6^{29}$ **(E)** $2 \times 6^{10}$

7. The average (mean) of a list of 10 numbers is 0. If 72 and $-12$ are added to the list, the new average will be

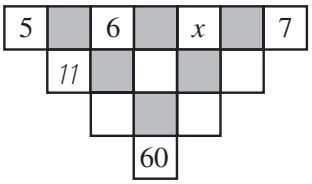
   **(A)** 30 **(B)** 6 **(C)** 0 **(D)** 60 **(E)** 5

8. On a rectangular table 5 units long and 2 units wide, a ball is rolled from point $P$ at an angle of 45° to $PQ$ and bounces off $SR$. The ball continues to bounce off the sides at 45° until it reaches $S$. How many bounces of the ball are required?

   **(A)** 9 **(B)** 8 **(C)** 7
   **(D)** 5 **(E)** 4

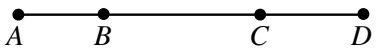


9. The number in an unshaded square is obtained by adding the numbers connected to it from the row above. (The '*11*' is one such number.) The value of $x$ must be




(**A**) 4 (**B**) 6 (**C**) 9
(**D**) 15 (**E**) 10

10. Four points are on a line segment, as shown.




If $AB:BC=1:2$ and $BC:CD=8:5$, then $AB:BD$ equals
(**A**) $4:13$ (**B**) $1:13$ (**C**) $1:7$
(**D**) $3:13$ (**E**) $4:17$

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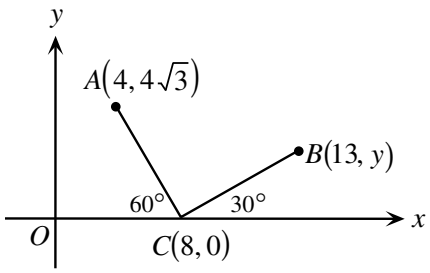
**Part B: Each question is worth 6 credits.**

11. The number of solutions $(x, y)$ of the equation $3x+y=100$, where $x$ and $y$ are positive integers, is
(**A**) 33 (**B**) 35 (**C**) 100 (**D**) 101 (**E**) 97

12. In the diagram, the value of $y$ is
(**A**) $\frac{13}{2\sqrt{3}}$ (**B**) $\frac{5}{\sqrt{3}}$ (**C**) 2
(**D**) 12 (**E**) $\frac{\sqrt{3}}{5}$




13. Three-digit integers are formed using only the digits 1 and/or 2. The sum of all such integers formed is
(**A**) 1332 (**B**) 333 (**C**) 999 (**D**) 666 (**E**) 1665

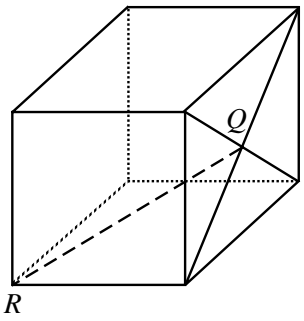
14. Three straight lines, $l_1$, $l_2$ and $l_3$, have slopes $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$, respectively. All three lines have the same $y$-intercept. If the sum of the $x$-intercepts of the three lines is 36, then the $y$-intercept is
(**A**) $\frac{-13}{12}$ (**B**) $\frac{-12}{13}$ (**C**) $-4$ (**D**) 4 (**E**) $-9$

15. If $-2 \le x \le 5$, $-3 \le y \le 7$, $4 \le z \le 8$, and $w = xy - z$, then the smallest value $w$ may have is
(**A**) $-14$ (**B**) $-18$ (**C**) $-19$ (**D**) $-22$ (**E**) $-23$

16. If $N = \left(7^{p+4}\right)\left(5^{q}\right)\left(2^{3}\right)$ is a perfect cube, where $p$ and $q$ are positive integers, the smallest possible value of $p+q$ is
(**A**) 5 (**B**) 2 (**C**) 8 (**D**) 6 (**E**) 12

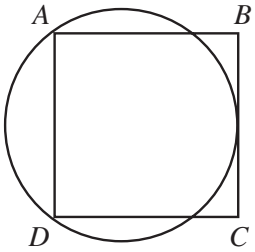
17. Using only digits 1, 2, 3, 4, and 5, a sequence is created as follows: one 1, two 2's, three 3's, four 4's, five 5's, six 1's, seven 2's, and so on.
The sequence appears as: 1, 2, 2, 3, 3, 3, 4, 4, 4, 4, 5, 5, 5, 5, 5, 1, 1, 1, 1, 1, 1, 2, 2, ... .
The 100th digit in the sequence is
(**A**) 1 (**B**) 2 (**C**) 3 (**D**) 4 (**E**) 5

18. $Q$ is the point of intersection of the diagonals of one face of a cube whose edges have length 2 units. The length of $QR$ is




(**A**) 2 (**B**) $\sqrt{8}$ (**C**) $\sqrt{5}$
(**D**) $\sqrt{12}$ (**E**) $\sqrt{6}$

19. Square $ABCD$ has sides of length 14. A circle is drawn through $A$ and $D$ so that it is tangent to $BC$, as shown. What is the radius of the circle?




(**A**) 8.5 (**B**) 8.75 (**C**) 9
(**D**) 9.25 (**E**) 9.5

20. A deck of 100 cards is numbered from 1 to 100. Each card has the same number printed on both sides. One side of each card is red and the other side is yellow. Barsby places all the cards, red side up, on a table. He first turns over every card that has a number divisible by 2. He then examines all the cards, and turns over every card that has a number divisible by 3. How many cards have the red side up when Barsby is finished?

(**A**) 83 (**B**) 17 (**C**) 66 (**D**) 50 (**E**) 49

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**Part C: Each question is worth 8 credits.**

21. The numbers 123 456 789 and 999 999 999 are multiplied. How many of the digits in the final result are 9's?

(**A**) 0 (**B**) 1 (**C**) 2 (**D**) 3 (**E**) 17

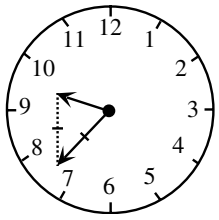
22. There are four unequal, positive integers $a$, $b$, $c$, and $N$ such that $N = 5a + 3b + 5c$. It is also true that $N = 4a + 5b + 4c$ and $N$ is between 131 and 150. What is the value of $a + b + c$?

(**A**) 13 (**B**) 17 (**C**) 22 (**D**) 33 (**E**) 36

23. Three rugs have a combined area of 200 $m^2$. By overlapping the rugs to cover a floor area of 140 $m^2$, the area which is covered by exactly two layers of rug is 24 $m^2$. What area of floor is covered by three layers of rug?

(**A**) 12 $m^2$ (**B**) 18 $m^2$ (**C**) 24 $m^2$ (**D**) 36 $m^2$ (**E**) 42 $m^2$

24. At some time between 9:30 and 10 o'clock the triangle determined by the minute hand and the hour hand is an isosceles triangle (see diagram). If the two equal angles in this triangle are each twice as large as the third angle, what is the time?




(**A**) 9:35 (**B**) 9:36 (**C**) 9:37
(**D**) 9:38 (**E**) 9:39

25. For each value of $x$, $f(x)$ is defined to be the minimum value of the three numbers $2x + 2$, $\frac{1}{2}x + 1$ and $-\frac{3}{4}x + 7$. What is the maximum value of $f(x)$?

(**A**) $\frac{2}{3}$ (**B**) 2 (**C**) $\frac{17}{5}$ (**D**) $\frac{62}{11}$ (**E**) 7