Canadian Mathematics Competition

An activity of The Centre for Education in Mathematics and Computing, University of Waterloo, Waterloo, Ontario

Cayley Contest (Grade 10)

Wednesday, February 18, 1998

Time: 1 hour

Calculators are permitted, providing they are non-programmable and without graphic displays.

Instructions

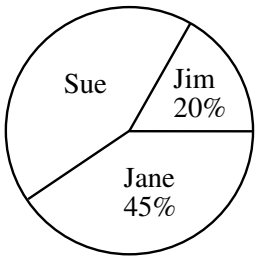
1. Do not open the contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name, city/town, and province in the box in the upper right corner.
5. **Be certain that you code your name, age, sex, grade, and the contest you are writing on the response form. Only those who do so can be counted as official contestants.**
6. This is a multiple-choice test. Each question is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. When you have decided on your choice, fill in the appropriate circles on the response form.
7. Scoring: Each correct answer is worth 5 credits in Part A, 6 credits in Part B, and 8 credits in Part C.
   There is *no penalty* for an incorrect answer.
   Each unanswered question is worth 2 credits, to a maximum of 20 credits.
8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor instructs you to begin, you will have *sixty* minutes of working time.

Scoring: There is *no penalty* for an incorrect answer.
Each unanswered question is worth 2 credits, to a maximum of 20 credits.

Part A: Each question is worth 5 credits.

1. The value of $(0.3)^2 + 0.1$ is
(**A**) 0.7 (**B**) 1 (**C**) 0.1 (**D**) 0.19 (**E**) 0.109

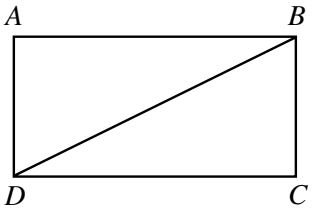
2. The pie chart shows a percentage breakdown of 1000 votes in a student election. How many votes did Sue receive?
(**A**) 550 (**B**) 350 (**C**) 330
(**D**) 450 (**E**) 935



3. The expression $\dfrac{a^9 \times a^{15}}{a^3}$ is equal to
(**A**) a^{45} (**B**) a^8 (**C**) a^{18} (**D**) a^{14} (**E**) a^{21}

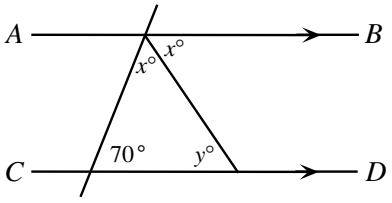
4. The product of two positive integers p and q is 100. What is the largest possible value of $p+q$?
(**A**) 52 (**B**) 101 (**C**) 20 (**D**) 29 (**E**) 25

5. In the diagram, $ABCD$ is a rectangle with $DC = 12$. If the area of triangle BDC is 30, what is the perimeter of rectangle $ABCD$?
(**A**) 34 (**B**) 44 (**C**) 30
(**D**) 29 (**E**) 60



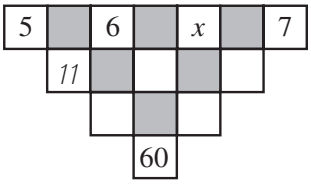
6. If $x = 2$ is a solution of the equation $qx - 3 = 11$, the value of q is
(**A**) 4 (**B**) 7 (**C**) 14 (**D**) -7 (**E**) -4

7. In the diagram, AB is parallel to CD. What is the value of y?
(**A**) 75 (**B**) 40 (**C**) 35
(**D**) 55 (**E**) 50



8. The vertices of a triangle have coordinates $(1,1)$, $(7,1)$ and $(5,3)$. What is the area of this triangle?
(**A**) 12 (**B**) 8 (**C**) 6 (**D**) 7 (**E**) 9

9. The number in an unshaded square is obtained by adding the numbers connected to it from the row above. (The '*11*' is one such number.) The value of x must be
(**A**) 4 (**B**) 6 (**C**) 9
(**D**) 15 (**E**) 10



10. The sum of the digits of a five-digit positive integer is 2. (A five-digit integer cannot start with zero.) The number of such integers is

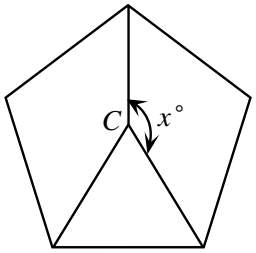
(**A**) 1 (**B**) 2 (**C**) 3 (**D**) 4 (**E**) 5

Part B: Each question is worth 6 credits.

11. If $x+y+z=25$, $x+y=19$ and $y+z=18$, then y equals

(**A**) 13 (**B**) 17 (**C**) 12 (**D**) 6 (**E**) -6

12. A regular pentagon with centre C is shown. The value of x is



(**A**) 144 (**B**) 150 (**C**) 120
(**D**) 108 (**E**) 72

13. If the surface area of a cube is 54, what is its volume?

(**A**) 36 (**B**) 9 (**C**) $\frac{81\sqrt{3}}{8}$ (**D**) 27 (**E**) $162\sqrt{6}$

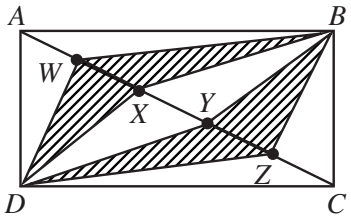
14. The number of solutions (x, y) of the equation $3x+y=100$, where x and y are positive integers, is

(**A**) 33 (**B**) 35 (**C**) 100 (**D**) 101 (**E**) 97

15. If $\sqrt{y-5}=5$ and $2^x=8$, then $x+y$ equals

(**A**) 13 (**B**) 28 (**C**) 33 (**D**) 35 (**E**) 38

16. Rectangle $ABCD$ has length 9 and width 5. Diagonal AC is divided into 5 equal parts at W, X, Y, and Z. Determine the area of the shaded region.



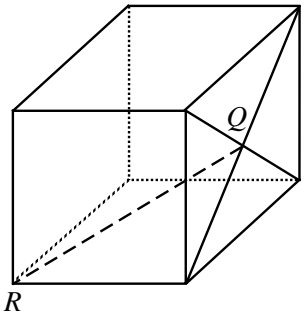
(**A**) 36 (**B**) $\frac{36}{5}$ (**C**) 18

(**D**) $\frac{4\sqrt{106}}{5}$ (**E**) $\frac{2\sqrt{106}}{5}$

17. If $N=\left(7^{p+4}\right)\left(5^q\right)\left(2^3\right)$ is a perfect cube, where p and q are positive integers, the smallest possible value of $p+q$ is

(**A**) 5 (**B**) 2 (**C**) 8 (**D**) 6 (**E**) 12

18. Q is the point of intersection of the diagonals of one face of a cube whose edges have length 2 units. The length of QR is

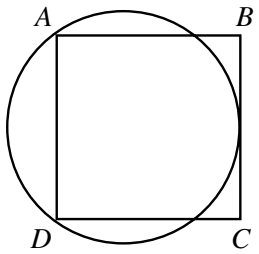


(**A**) 2 (**B**) $\sqrt{8}$ (**C**) $\sqrt{5}$
(**D**) $\sqrt{12}$ (**E**) $\sqrt{6}$

19. Mr. Anderson has more than 25 students in his class. He has more than 2 but fewer than 10 boys and more than 14 but fewer than 23 girls in his class. How many different class *sizes* would satisfy these conditions?

(**A**) 5 (**B**) 6 (**C**) 7 (**D**) 3 (**E**) 4

20. Each side of square $ABCD$ is 8. A circle is drawn through A and D so that it is tangent to BC. What is the radius of this circle?



(**A**) 4 (**B**) 5 (**C**) 6

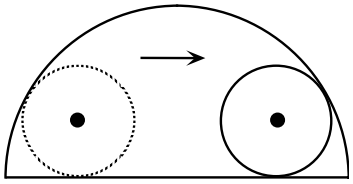
(**D**) $4\sqrt{2}$ (**E**) 5.25

Part C: Each question is worth 8 credits.

21. When Betty substitutes $x = 1$ into the expression $ax^3 - 2x + c$ its value is -5. When she substitutes $x = 4$ the expression has value 52. One value of x that makes the expression equal to zero is

(**A**) 2 (**B**) $\frac{5}{2}$ (**C**) 3 (**D**) $\frac{7}{2}$ (**E**) 4

22. A wheel of radius 8 rolls along the diameter of a semicircle of radius 25 until it bumps into this semicircle. What is the length of the portion of the diameter that cannot be touched by the wheel?



(**A**) 8 (**B**) 12 (**C**) 15

(**D**) 17 (**E**) 20

23. There are four unequal, positive integers a, b, c, and N such that $N = 5a + 3b + 5c$. It is also true that $N = 4a + 5b + 4c$ and N is between 131 and 150. What is the value of $a + b + c$?

(**A**) 13 (**B**) 17 (**C**) 22 (**D**) 33 (**E**) 36

24. Three rugs have a combined area of 200 m^2. By overlapping the rugs to cover a floor area of 140 m^2, the area which is covered by exactly two layers of rug is 24 m^2. What area of floor is covered by three layers of rug?

(**A**) 12 m^2 (**B**) 18 m^2 (**C**) 24 m^2 (**D**) 36 m^2 (**E**) 42 m^2

25. One way to pack a 100 by 100 square with 10 000 circles, each of diameter 1, is to put them in 100 rows with 100 circles in each row. If the circles are repacked so that the centres of any three tangent circles form an equilateral triangle, what is the maximum number of additional circles that can be packed?

(**A**) 647 (**B**) 1442 (**C**) 1343 (**D**) 1443 (**E**) 1344